Exponential Function

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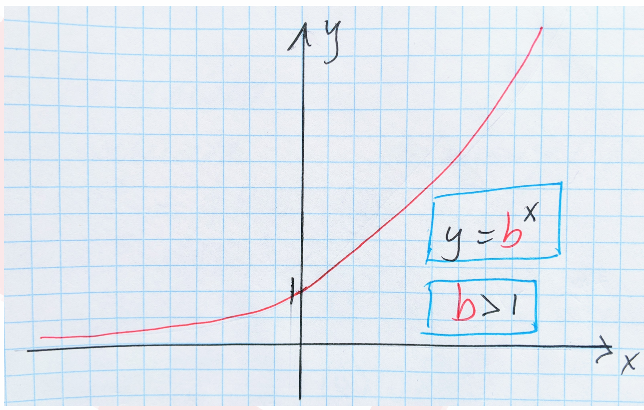
Exponentials

The exponential function is a function with the following form,

$$\boxed{f(x) = b^x}$$

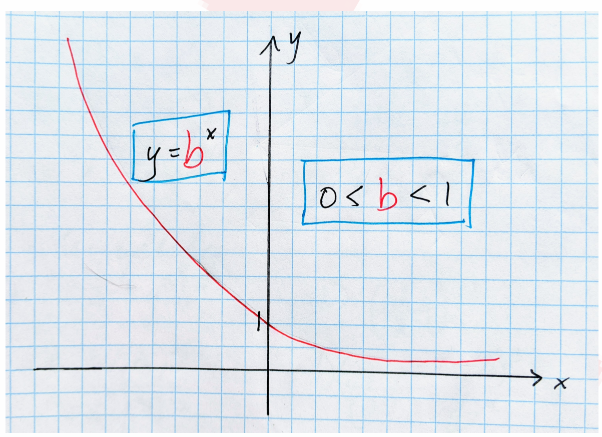
where b is a constant real number, $b \in \mathbb{R}$, and $x \in \mathbb{R}$.

Depending on the value of b, the graph of the exponential function will change. Let's have a look. When, $b > 1$ we have the following graph of $y = f(x) = b^x$:



We can say that the function is "increasing".

When $0 < b < 1$ the graph of $y = f(x) = b^x$ has the following form:



We can say the function is "decreasing".

Notice in both situations above, the graph of the function approaches but doesn't touch or cross the x-axis. The x-axis for the above exponential functions is an *asymptote*. What is an asymptote? An *asympote* is a line that a funcion approaches but never reaches and thus never crosses. Let's summarize the properties of the exponential function below.

Properties of the exponential function $y = b^x$

1. $b > 0$
2. y-intercept =1
3. x-axis is the horizontal aymptote.
4. $domain = \{x | x \in \mathbb{R}\}$
5. $range = \{y \in \mathbb{R} | y > 0\}$
6. $y = b^x$ is increasing if $b > 1$
7. $y = b^x$ is decreasing if $0 < b < 1$

Now that we know what an exponential function looks like and have looked at the parent function $f(x) = b^x$, let's review the transformation and what they look like when applied to an exponential function.

Transformation of the exponential function $y = b^x$

$$\boxed{f(x) = ab^{k(x-d)} + c}$$

The parent function is b^x.

$c =$ vertical translation
$c < 0$ vertical translation down
$c > 0$ veritcal translation up
$d =$ horizontal translation
$d > 0$ horizontal translation left
$d < 0$ horizontal translation right
$k =$ horizonal stretch or compression
$k > 1$ is a horiztonal compression
$0 < k < 1$ is a horizontal stretch
$k < 0$ horizontal reflection
$a =$ vertical stretch or compression
$a > 1$ is a vertical stretch
$0 < a < 1$ is a vertical compression
$a < 0$ vertical reflection

Exercises

For each function in below state,

i) the parent function

ii) the transformations applied to the base functions

iii) the equation of the horizontal asymptote.

iv) the y-intercept

v) the x-intercept, if there is one

vi) the domain

vii) the range.

(a) 3^x

(b) $\left(\frac{1}{4}\right)^x$

(c) $3(4)^x + 2$

(d) $-2^x + 1$

(e) $3\left(\frac{1}{2}\right)^x$

(f) $-\left(\frac{1}{5}\right)^x + 3$

(g) $2(3^x) - 1$

(h) $-6\left(\frac{1}{3}\right)^x + 5$

(i) $7^x - 4$

(j) $\left(\frac{1}{8}\right)^x + 1$