Exponential Growth and Decay

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Exponential growth and decay

This is a common application of the exponential function. Exponential growth/decay occurs when quantitites increase or decrease at a rate proportional to the quantit present. Some examples of where growth or decay occurs is in savings accounts, size of populations, decay of radioactive chemicals. Let's look at an example.

Example

The population of a city is 810 000. If it is increasing at 4% per year, estimate the population in four years.

Solution:

$$y = C(1 + 0.04)^t$$

$C = 810000 =$ initial population; $y =$ population after t years. Therefore we have,

$$\begin{aligned} y &= 810000(1.04)^t \\ y(4) &= 810000(1.04)^4 \\ &= 947585.4 \end{aligned}$$

So the population after 4 years is approximately 947586.

Example

A used car dealer sells a five year old car for \$4200. What was the original value of the car if the depreciation is 15% a year?

Solution:

$$y = Cb^t$$

where $b = 1 - 0.15 = 0.85$

$$\begin{aligned} y(5) = 4200 &= C(0.85)^5 \\ \frac{4200}{(085)^5} &= C \\ \$9465.74 &= C \end{aligned}$$

Therefore, the original price of the car is \$9465.74.

Example

A bacteria population doubles in 5 days. When will it be 16 times as large?

Solution:

$$y = C2^{t/5}$$

where C = initial population and y = population after t days.

$$\begin{aligned} y &= C2^{t/5} \\ \frac{16C}{C} &= \frac{C}{C}2^{t/5} \\ 16 &= 2^{t/5} \\ 2^4 &= 2^{t/5} \\ 4 &= t/5 \\ 20 &= t \end{aligned}$$

Therefore, after 20 days the population will be 16 times as great as the initial population.

Example

A research assistant made 160mg of radioactive sodium Na^{24} and found that there was only 20mg left after 45 hours. What is the half life of Na^{24}?

Solution: We have $C = 160$, '$y(45) = 20$ and we want to find $b = ?$.

$$\begin{aligned}
y(t) &= Cb^t \\
y(t) &= C\left(\frac{1}{2}\right)^t \\
20 &= 160\left(\frac{1}{2}\right)^{45k} \\
\frac{20}{160} &= \frac{160}{160}\left(\frac{1}{2}\right)^{45k} \\
\frac{1}{8} &= \left(\frac{1}{2}\right)^{45k} \\
\frac{1}{2^3} &= \left(\frac{1}{2}\right)^{45k} \\
\frac{3}{45} &= \frac{45k}{45} \\
\frac{1}{15} &= k
\end{aligned}$$

Therefore,

$$\begin{aligned}
y(t) &= 160\left(\frac{1}{2}\right)^{t/15} \\
\frac{80}{160} &= \frac{160}{160}\left(\frac{1}{2}\right)^{t/15} \\
\frac{1}{2} &= \left(\frac{1}{2}\right)^{t/15} \\
1 &= \frac{t}{15} \\
15 &= t
\end{aligned}$$

Therefore, the half life is 15 hours.

Exercises

1. For the following funcations state and/or find the following,

   i Does the function represent growth or decay?

   ii What is the growth or decay rate?

   iii What is the initial value?

   (a) $y = 1200(1.3)^t$

   (b) $y = 55(0.8)^t$

   (c) $y = 100(1.25)^t$

   (d) $y = 200(1.05)^t$

   (e) $y = 14000(0.92)^t$

   (f) $y = 225(0.1)^t$

   (g) $y = 10\left(\frac{2}{3}\right)^t$

   (h) $y = 50(1.15)^x$

   (i) $y = 85(0.65)^x$

   (j) $y = 6000(1.12)^x$

2. For those functions in # 1 that represent *decay*, find the *half-life*.