Graph Functions
Parent Funtions

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2021

Graphing Functions

You have probably looked at the graphs of quadratics of of the quadratic function,

$$f(x) = ax^2 + bx + c$$

Here we will consider the graph of a general function f. This will include considering the various transformation and how they affect a general function. For a function f that has been transformed as follows:

$$y = af(k(x - d)) + c \tag{1}$$

we have the following transformations:

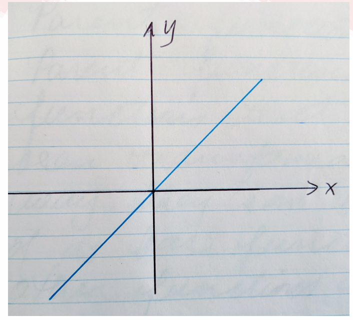
Value	Description of transformation
$c > 0$	Vertical translations upwards c units
$c < 0$	Vertical translation downwards c units
$d > 0$	Horizontal translation right d units
$d < 0$	Horizontal translation left d units
$a > 1$	Vertical stretch by a factor of a
$0 < a < 1$	Vertical compression by a factor of $1/a$
$-1 < a < 0$	Reflection in the x-axis & a vertical compression by a factor of $1/\|a\|$
$a < -1$	Reflection in the x-axis and vertical stretch by $\|a\|$
$1 < k$	Horizontal compression by a factor of $1/k$
$0 < k < 1$	Horizontal stretch by k
$-1 < k < 0$	Reflection in the y-axis & a horiztonal stretch by a factor of k
$k < -1$	Reflection in the y-axis & a horizontal compression by a factor of $1/k$

When graphing a quadratic function, all the transformations were in relation to a base quadratic x^2. In this same way, for any given function f, before we can determine what transformations have occured, we have to determine what this base function is or *parent funtion* is. This is the function that has no transformations applied to it.

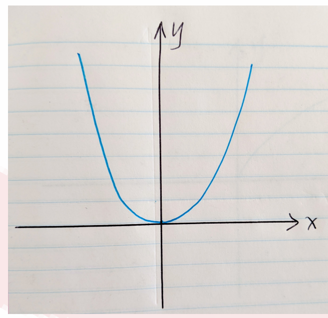
Parent Functions

Parent functions are functions that haven't been transformed in any way. They can be thought of as base functions all other functions are comprised of through various combinations or additions, subtraction, multiplication, division and composition. Let's have a look at some of the more common base functions.

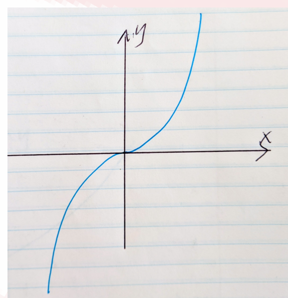
$f(x) = x$



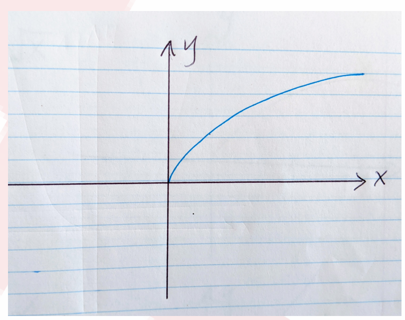
$f(x) = x^2$



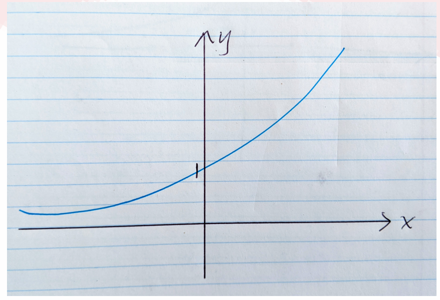
$f(x) = x^3$



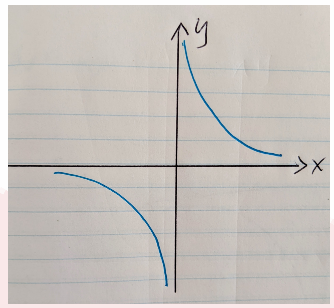
$f(x) = \sqrt{x}$



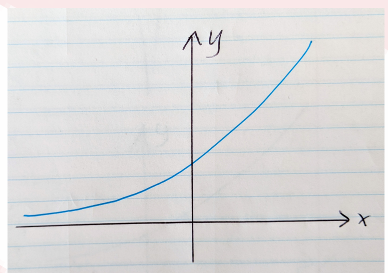
$$f(x) = 2^x$$



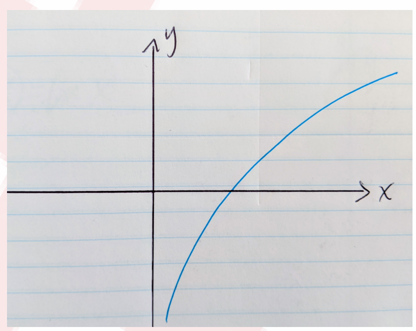
$$f(x) = \frac{1}{x}$$



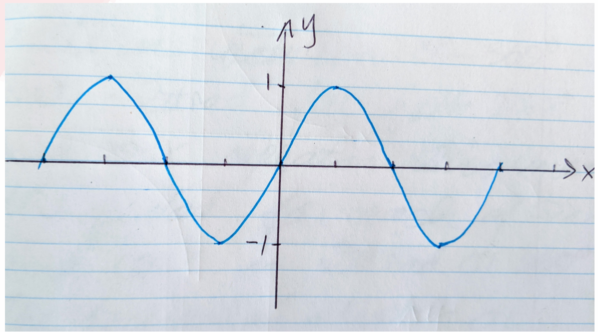
$$f(x) = e^x$$



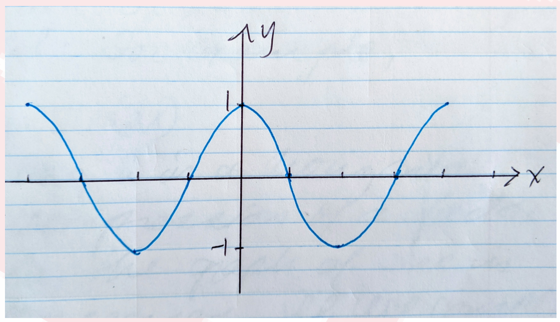
$$f(x) = \ln x$$



$f(x) = \sin x$



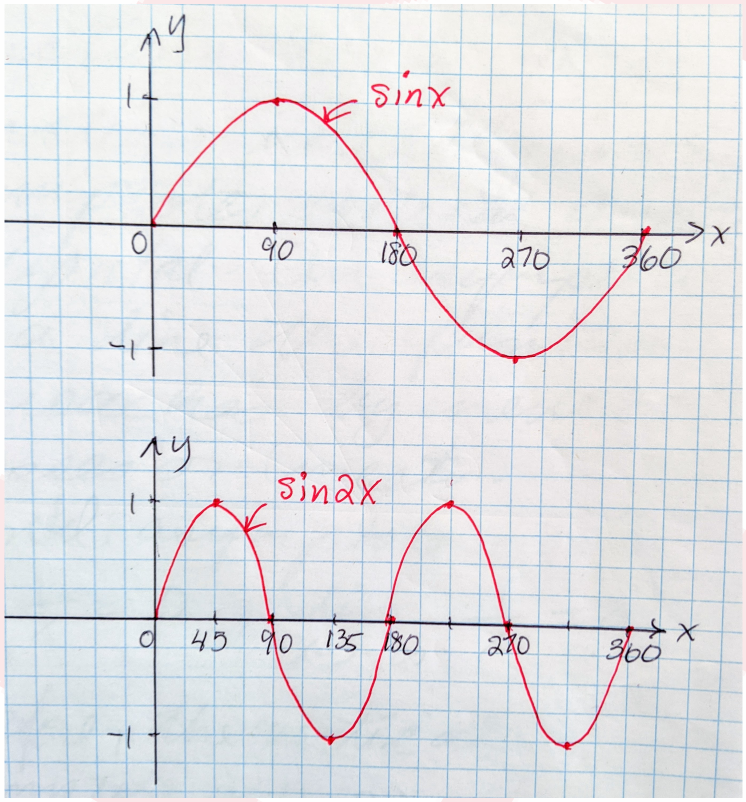
$f(x) = \cos x$



The last two functions $f(x) = \sin x$ and $f(x) = \cos x$ are trigonometric functions and are *periodic*. This means that the values and graph of the function repeats itself. If you notice the pattern betwen $0° - 360°$ or 0 radians - 2π radians for both $\sin x$ and $\cos x$ repeats. This means the period for each of $\sin x$ and $\cos x$ is 2π radians or $360°$. In this situation the horizontal stretch or compression factor will change the period when $\sin x$ or $\cos x$ are transformed. For example,

$$y = \sin(2x)$$

The 2 is a horizontal compression of a factor of $\frac{1}{2}$, So the graph of $\sin x$ is compressed. If we consider one period of $\sin x$ we have the following,



Notice that the period has been divided by two. From here we can infer that the period of the sine function (and cosine function) when transformed as follows,

$$y = a\sin(k(x-d)) + c$$

is given by,

$$\text{new period} = \frac{2\pi}{k}$$

Details on graphs of trigonmetric functions will be considered in another worksheet.

Exercises

For the following functions, determine,

i) the parent function,

ii) list the transformations

iii) graph the functions

a) $f(x) = \frac{1}{-x+2} + 2$

b) $f(x) = -3x + 4$

c) $f(x) = \frac{1}{2}(x-1)^3 + 2$

d) $f(x) = 2\sin(x-45) + 1$

e) $f(x) = -\frac{1}{2}\cos(3x + 90) + 2$