Factoring Polynomials

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Factoring polynomials

Let's considering factoring single variable polynomials. We'll start with the following example.

$$4x^3 + x^2$$

Steps to consider when factoring polynomials.

1. **Is there a common fator in each term?** Yes. x^2.

$$4x^3 + x^2$$

2. **Factor that common term out.**

$$\begin{aligned} & 4x^3 + x^2 \\ = \ & x^2(4x + 1) \end{aligned}$$

3. **Can each of the factors be written as a factor of polynomials of smaller degree.** No
4. **Now we're done factoring.**

$$x^2(4x + 1)$$

is the *factored* form of $4x^3 + x^2$.

Let's consider another example.

Example

Factor the following polynomial,

$$ax + 2x + ay + 2y$$

In this example we're going to consider the terms with $x's$ separately from the terms with the $y's$, as the first step.

1. **What are the different variables in the polynomial?** In this polynomial we have the variables x and y. a is a constant term.

2. **Group all terms with $x's$ together and all terms with $y's$ together.**
$$(ax + 2x) + (ay + 2y)$$

3. **Is there a common factor in each term, for the $x's$ and then for the $y's$, independently of the other?** Yes.
$$\begin{aligned} & (ax + 2x) + (ay + 2y) \\ = \; & x(a + 2) + y(a + 2) \end{aligned}$$

4. **Is there a common factor in each term?** Yes. The common factor in this case is $(a + 2)$.
$$\begin{aligned} & (ax + 2x) + (ay + 2y) \\ = \; & x(a + 2) + y(a + 2) \\ = \; & (x + y)(a + 2) \end{aligned}$$

5. **Can we write any of the factors as polynomials of lesser degree?** No.

6. **Now we've completely factored our polynomial.**
$$(x + y)(a + 2)$$
is the factored form of $ax + 2x + ay + 2y$.

Exercises

Factor the following,

a) $a^4 + 4a^2 + 4a^3$

b) $6x^3 + 2x^2$

c) $-8y^9 + y^7$

d) $x^2 + x^4$

e) $3x^3 + x^9 + x^2$

f) $ax - 4x - 4y + ay$

g) $4b + 4c + 4d + xb + xc + xd$

h) $ax + bx + bz + az$

i) $xyz + 3yz - xz - 3z$

j) $-6x + cy - 6y + cx$