Addition and Subtraction Formulas

RaiseMyMarks.com

2021

Addition and Subraction formulas

The triangles and related special angles, $\pi/3,\ \pi/6, \pi/4$ have particular values for each of the three trigonometric ratios, $\sin x,\ \cos x,\ \tan x$. If a given angle can be written as the sum or difference of any of these special angles, using the addition and subtraction formulas below it is possible to find exact values to the trigonometric ratios for some angles. Let's first have a look at the the addition and subtraction formulas then we'll take a look at some examples.

Addition and subraction formuals for trigonometric functions

$$\sin(x+y) = \sin x\cos y + \cos x\sin y \tag{1}$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y \tag{2}$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y \tag{3}$$
$$\cos(x-y) = \cos x\cos y + \sin x\sin y \tag{4}$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y} \tag{5}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y} \tag{6}$$

Double Angle Formulas

$$\tan(2x) = \frac{2\tan x}{1-\tan^2 x} \tag{7}$$
$$\cos(2x) = \cos^2 x - \sin^2 x \tag{8}$$
$$\sin(2x) = 2\sin x\cos x \tag{9}$$
$$\sin^2 x = \frac{1-\cos 2x}{2} \tag{10}$$
$$\cos^2 x = \frac{1+\cos 2x}{2} \tag{11}$$

Example

Find the exact value of $\tan 75°$.

Solution:

$$\begin{aligned}
\tan(75°) &= \tan(45° + 30°) \\
&= \frac{\tan(45°) + \tan(30°)}{1 - \tan(45°)\tan(30°)} \\
&= \frac{1 + 1/\sqrt{3}}{1 - (1)(1/\sqrt{3})} \\
&= \frac{(\sqrt{3}+1)\sqrt{3}}{(\sqrt{3}-1)\sqrt{3}} \\
&= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) \\
&= \frac{3 + 2\sqrt{3} + 1}{3 - 1} \\
&= \frac{4 + 2\sqrt{3}}{2} \\
&= 2 + \sqrt{3}
\end{aligned}$$

Therefore, $\tan(75°) = 2\sqrt{3}$.

Example

Show $\tan(180 - x) = -\tan x$

Solution:

$$\begin{aligned} L.S. &= \tan(180 - x) \\ &= \frac{\tan(180) - \tan x}{1 + \tan(180)\tan x} \\ &= \frac{0 - \tan x}{1 + (0)\tan x} \\ &= -\frac{\tan x}{1} \\ &= -\tan x \\ &= R.S. \end{aligned}$$

Example

Find $\sin\left(\frac{7\pi}{12}\right)$

Solution: First let's try and rewrite the angle $7\pi/12$ as the sum or difference of the special angles.

$$\frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi + 4\pi}{12} = \frac{7\pi}{12}$$

or in degrees,

$$\frac{7\pi}{12} = \frac{7(180)}{12} = 7 \times 15 = 75^\circ = 45^\circ + 30^\circ$$

Now, le'ts apply one of the addition formulas and see what value we get applying the special triangle/angle values.

$$\begin{aligned}
\sin\left(\frac{7\pi}{12}\right) &= \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right) \\
&= \sin\frac{\pi}{4}\cos\frac{\pi}{3}+\cos\frac{\pi}{4}\sin\frac{\pi}{3} \\
&= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) \\
&= \left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{(1+\sqrt{3})\sqrt{2}}{2(\sqrt{2})} \\
&= \frac{(1+\sqrt{3})\sqrt{2}}{4}
\end{aligned}$$

Example

Show $\sin 2\theta = 2\sin\theta\cos\theta$

Solution:

$$\begin{aligned}
L.S. &= \sin 2\theta \\
&= \sin(\theta+\theta) \\
&= \sin\theta\cos\theta+\cos\theta\sin\theta \\
&= 2\sin\theta\cos\theta \\
&= R.S.
\end{aligned}$$

Example

Given $\tan A = -3$ find $\tan(2A)$.

Solution:

$$\begin{aligned}
\tan(2A) &= \frac{2\tan A}{1-\tan^2 A} \\
&= \frac{2(-3)}{1-(-3)^2} \\
&= \frac{-6}{1-9} \\
&= \frac{-6}{-8} \\
&= \frac{3}{4}
\end{aligned}$$

Exercises

1. Find the exact value of,
   (a) sin(150)

   (b) tan(225)

   (c) cos(225)

   (d) sin(225)

   (e) tan(135)

   (f) cos(135)

(g) $\sin(135)$

(h) $\tan(270)$

(i) $\cos(270)$

(j) $\sin(270)$

