

Solving Trigonometric Equations

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Recall solving an equation of the the following form,

$$x^2 - 3x - 4 = 0 \quad (1)$$

The goal when solving the above equation is to find the values for x that satisfy the equation or in other words, when the value for x is plugged into the left hand side of (1), it equals the right hand side of (1). The first step when solving an equation of the form in (1) is to *factor* the left hand side. Let's do this.

$$LHS = x^2 - 4x + 4 = (x - 4)(x + 1) \quad (2)$$

Next, when does a product equal zero? A product equals zero when at least one factor equals zero. So in our case (2) equals zero when,

$$x - 4 = 0 \text{ or} \quad (3)$$

$$x + 1 = 0 \quad (4)$$

Solving (3) and (4) for x we get,

$$x = 4 \text{ or } x = -1$$

So the solutions to the equation in (1) are $x = 4$ or $x = -1$. The above thought process holds for every equation. Let's consider another example.

Example

Solve the following equation,

$$\sin x + 2 = 3 \quad (5)$$

Solution: We need to find the values of x for which the equation (5) is satisfied. We are used to working with trigonometric functions equal to a single number such as,

$$\sin \theta = 1.$$

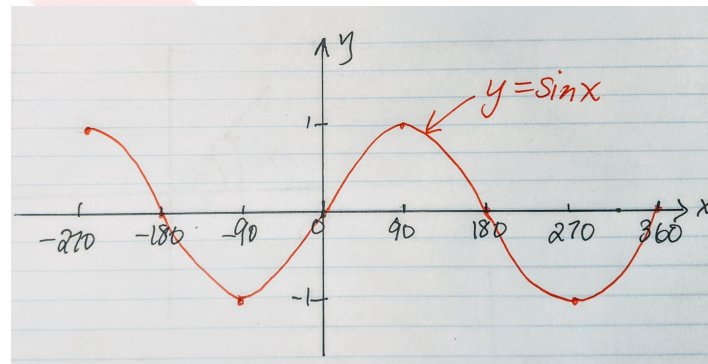
Let's see if we can rearrange equation (5) so we have something of this form, trigonometric function on one side of the equal sign and number on the other.

$$\sin x + 2 = 3$$

$$\sin x = 3 - 2$$

$$\sin x = 1$$

What angle values have a sine of 1? We can use the graph of sine if we don't already know what angle values,



We will consider the angle values between 0° and 360° since the sine function is a periodic function with period 360° . From the graph we see that at the angle value 90° , $\sin 90^\circ = 1$. Therefore, when $x = 90^\circ$, $\sin x = 1$. Therefore, our solution to the trigonometric function $\sin x + 2 = 3$ is $x = 90^\circ$.

Exercises

Solve the following equations,

(a) $\cot\left(\frac{x}{2}\right) = 0$

(b) $\cos\frac{x}{4} = -1$

(c) $\sin\left(\frac{x}{3} + 45^\circ\right) = \frac{\sqrt{2}}{2}$

(d) $\cot(2x - 1) = 1$

(e) $\csc x = \frac{2}{\sqrt{3}}$

$$(f) \tan^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} = 0$$

$$(g) 4 \sin^3 x + 2 \sin^2 x - 2 \sin x - 1 = 0$$