

Trigonometric Identities

**Raise My**  
**MArks**

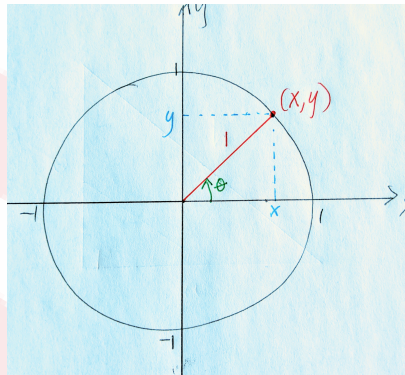
RaiseMyMarks.com

2021

## Trigonometric Identities

Trigonometric identities are relationships between our three trigonometric functions  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ . The addition and subtraction formulas are trigonometric identities. Below are some more trigonometric identities. We'll start with the Pythagorean Identity.

### Pythagorean Identity



From the diagram above we can see that we have the following values of the trigonometric ratios,

$$\sin \theta = \frac{y}{1} = y$$
$$\cos \theta = \frac{x}{1} = x$$

We take a point on the unit circle with coordinates  $(x, y)$  and now are able to rewrite the point in terms of the angle created from the positive x-axis and the line segment from the origin to the given point on the unit circle  $(x, y)$ . Namely, we have

$$(x, y) = (\cos \theta, \sin \theta)$$

We also know that a point  $(x, y)$  on a circle of radius  $r$  satisfies the equation,

$$x^2 + y^2 = r^2$$

In the above situation if we let,

$$r = 1, \quad x = \cos \theta, \quad y = \sin \theta$$

then we have,

$$\cos^2 \theta + \sin^2 \theta = 1, \quad \text{or} \quad (1)$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (2)$$

If we divide equation (1) by  $\sin^2 \theta$  and divide equation (2) by  $\cos^2 \theta$  we have,

$$\cot^2 \theta + 1 = \csc^2 \theta, \quad \text{and} \quad (3)$$

$$\tan^2 \theta + 1 = \sec^2 \theta \quad (4)$$

### Half Angle Formulas

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

**Cofunction Identities**

$$\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right) = -\cos\left(\theta + \frac{\pi}{2}\right)$$

$$\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right) = \sin\left(\theta + \frac{\pi}{2}\right)$$

$$\tan \theta = \cot\left(\frac{\pi}{2} - \theta\right) = -\cot\left(\theta + \frac{\pi}{2}\right)$$

$$\cot \theta = \tan\left(\frac{\pi}{2} - \theta\right) = -\tan\left(\theta + \frac{\pi}{2}\right)$$

$$\csc \theta = \sec\left(\frac{\pi}{2} - \theta\right) = -\sec\left(\theta + \frac{\pi}{2}\right)$$

$$\sec \theta = \csc\left(\frac{\pi}{2} - \theta\right) = \csc\left(\theta + \frac{\pi}{2}\right)$$

**Reciprocal Identities**

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

**Exercises**

Simplify the fractions.

(a)

$$\frac{1 + \tan^2 x}{2 + \cot^2 x}$$

(b)

$$\frac{\sec^2 a - \cos^2 a}{\tan^2 a}$$

(c)

$$\frac{\csc^2 a - \sin^2 a}{\csc^2(2 - \cos^2 a)}$$

(d)

$$\frac{\sin^2 x}{1 + \cos x}$$

(e)

$$\frac{\sin 2a}{1 - \cos^2 a} \frac{\sin 2a}{\cos a}$$

(f)

$$\frac{\sin 3a - \sin 5a}{\cos 3a + \cos 5a}$$