Trigonometric Identities

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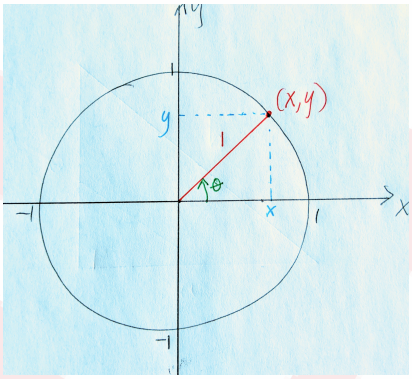
Trigonometric Identities

Trigonometric identities are relationships between our three trigonometric functions $\sin\theta$, $\cos\theta$ and $\tan\theta$. The addition and subtraction formulas are trigonometric identities. Below are some more trigonometric identities. We'll start with the Pythagorean Identity.

Pythagorean Identity



From the diagram above we can see that we have the following values of the trigonometric ratios,

$$\begin{aligned}\sin\theta &= \frac{y}{1} = y\\ \cos\theta &= \frac{x}{1} = x\end{aligned}$$

We take a point on the unit circle with coordinates (x, y) and now are able to rewrite the point in terms of the angle created from the positive x-axis and the line segment from the origin to the given point on the unit circle (x, y). Namely, we have

$$(x, y) = (\cos\theta, \sin\theta)$$

We also know that a point (x, y) on a circle of radius r satisfies the equation,

$$x^2 + y^2 = r^2$$

In the above situation if we let,

$$r = 1,\ x = \cos\theta,\ y = \sin\theta$$

then we have,

$$\cos^2\theta + \sin^2\theta = 1, \quad \text{or} \tag{1}$$
$$\sin^2\theta + \cos^2\theta = 1 \tag{2}$$

If we divide equation (1) by $\sin^2\theta$ and divide equation (2) by $\cos^2\theta$ we have,

$$\cot^2\theta + 1 = \csc^2\theta, \quad \text{and} \tag{3}$$
$$\tan^2\theta + 1 = \sec^2\theta \tag{4}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$
$$\cos^\theta = \frac{1+\cos(2\theta)}{2}$$
$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Cofunction Identities

$$\begin{aligned}
\sin\theta &= \cos(\frac{\pi}{2}-\theta) = -\cos(\theta+\frac{\pi}{2}) \\
\cos\theta &= \sin(\frac{\pi}{2}-\theta) = \sin(\theta+\frac{\pi}{2}) \\
\tan\theta &= \cot(\frac{\pi}{2}-\theta) = -\cot(\theta+\frac{\pi}{2}) \\
\cot\theta &= \tan(\frac{\pi}{2}-\theta) = -\tan(\theta+\frac{\pi}{2}) \\
\csc\theta &= \sec(\frac{\pi}{2}-\theta) = -\sec(\theta+\frac{\pi}{2}) \\
\sec\theta &= \csc(\frac{\pi}{2}-\theta) = \csc(\theta+\frac{\pi}{2})
\end{aligned}$$

Reciprocal Identities

$$\begin{aligned}
\csc\theta &= \frac{1}{\sin\theta} \\
\sec\theta &= \frac{1}{\cos\theta} \\
\cot\theta &= \frac{1}{\tan\theta}
\end{aligned}$$

Exercises

Prove the following identities.

(a) $\tan\alpha + \cot\alpha = \sec\alpha + \csc\alpha$

(b) $\cot^2 x = \cos^2 x + (\cot x \cos x)^2$

(c) $\frac{1}{\sec^2 a} = \sin^2 a \cos^2 a + \cos^4 a$

(d) $\cot a \sec a = \csc a$

(e) $\sec^2 a + \csc a = \frac{1}{\sin^2 a \cos^2 a}$

(f) $\sin b \cos(a-b) + \cos b \sin(a-b) = \sin a$

(g) $\cot(a+b) = \frac{\cot a \cot b - 1}{\cot a + \cot b}$

