

Arc Length

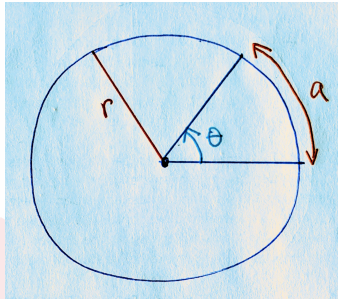
Raise My
Marks

RaiseMyMarks.com

2021

Arc Length

What is **arc length**? The arc length is the length of the arc travelled around a circle with a given radius r for a given angle θ in radians. Let's consider the following diagram of a circle with radius r .



A slice of pie or *sector* of the circle is cut out. The angle at the centre of the circle of the pie or setor is θ and the length of the outer edge or arc length of the sector is given by a . The relationship between θ , a and r is given by,

$$\theta = \frac{a}{r} \quad (1)$$

Let's consider an example.

Example

Find the arc length of a sector of a circle with radius $r = 9\text{cm}$ with angle 45° .

Solution: The angle 45° can be written as $\pi/4$ radians. Using equation (1) we can solve for a ,

$$\begin{aligned} \theta &= \frac{a}{r} \\ a &= r\theta \end{aligned}$$

Now we can plug in our values for θ and r and evaluate for a .

$$\begin{aligned} a &= (9)(\pi/4) \\ &= \frac{9\pi}{4} \end{aligned}$$

Therefore, the arclength of the circle of radius 9cm with angle 45° is $\frac{9\pi}{4}\text{cm}$.

Exercises

Find the arc length travelled throughout the angle θ below along a circle with the radius r given below.

(a) $\theta = \frac{7\pi}{6}$, $r = 3$

(b) $\theta = \frac{3\pi}{4}$, $r = 4$

(c) $\theta = \frac{\pi}{4}$, $r = 2$

(d) $\theta = \frac{\pi}{2}$, $r = 5$

(e) $\theta = \frac{\pi}{5}$, $r = 6$

(f) $\theta = \frac{4\pi}{3}$, $r = 2$

(g) $\theta = 2\pi$, $r = 5$

(h) $\theta = \frac{6\pi}{11}$, $r = 3$

(i) $\theta = \pi$, $r = 8$

(j) $\theta = \frac{5\pi}{7}$, $r = 9$