Geometric Series

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Geometric Series

A geometric series is the sum of the terms in a geometric sequence. Let's consider the geometric sequences and then sums.

$$\sum_1 = 3 + 6 + 12 + 24 + 48 + 96 + \cdots$$
$$\sum_2 = 2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \cdots$$
$$\sum_3 = 2 - 2 + 2 - 2 + 2 - 2 + \cdots$$

Let's consder the sum of the first n terms of the series. We will let the general temr of the series be denoted by t_i. This general term is the same as the general term for a geometric sequence. So, $t_i = ar^i,\ i = 0, 1, 2, ldots$. Now the sum of the first n terms is,

$$\begin{aligned} S_n &= t_0 + t_1 + \cdots + t_n \\ &= a + nar + ar^2 + \cdots + ar^n \\ &= a(1 + r + r^2 + \cdots + r^n) \\ &= a\left(\frac{1 - r^n}{1 - r}\right) \end{aligned}$$

Note: $1 + r + r^2 + \cdots + r^n = \frac{1-r^n}{1-r},\ r \neq 1$. So,

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \text{ or} \quad (1)$$
$$S_n = \frac{a - t_n}{1 - r} \quad (2)$$

Exercises

1. Given the information below, find a_3, a_6 and a_9 for the geometric sequence,

   a) $a_1 = 8,\ a_4 = 320$

   b) $a_4 = -\frac{1}{9},\ r = \frac{1}{3}$

   c) $a_5 = 48,\ a_6 = 96$

   d) $a_4 = \frac{1}{9},\ r = -\frac{1}{3}$

   e) $a_7 = 729,\ r = 3$

   f) $a_3 = 18,\ a_5 = 162$

   g) $a_7 = -2,\ r = -\frac{1}{3}$

   h) $a_4 - a_2 = 8,\ a_2 + a_3 = 4$

   i) $a_3 = -1,\ a_7 = \frac{1}{16}$

   j) $a_5 = 4a_3,\ a_3 + a_0 = 63$

