Geometric Series

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Geometric Series

A geometric series is the sum of the terms in a geometric sequence. Let's consider the geometric sequences and then sums.

$$\sum_1 = 3 + 6 + 12 + 24 + 48 + 96 + \cdots$$

$$\sum_2 = 2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \cdots$$

$$\sum_3 = 2 - 2 + 2 - 2 + 2 - 2 + \cdots$$

Let's consder the sum of the first n terms of the series. We will let the general temr of the series be denoted by t_i. This general term is the same as the general term for a geometric sequence. So, $t_i = ar^i$, $i = 0, 1, 2, ldots$. Now the sum of the first n terms is,

$$
\begin{aligned}
S_n &= t_0 + t_1 + \cdots + t_n \\
&= a + nar + ar^2 + \cdots + ar^n \\
&= a(1 + r + r^2 + \cdots + r^n) \\
&= a\left(\frac{1 - r^n}{1 - r}\right)
\end{aligned}
$$

Note: $1 + r + r^2 + \cdots + r^n = \frac{1-r^n}{1-r}$, $r \neq 1$. So,

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \text{ or} \tag{1}$$

$$S_n = \frac{a - t_n}{1 - r} \tag{2}$$

Exercises

1. Determine which series are arithmetic.

   a) $1 + 2 + 3 + \cdots$

   b) $3 + 5 + 7 + 9 + 11 + \cdots$

   c) $1 + 2 + 4 + 8 + 16 + \cdots$

   d) $1 + 3/2 + 2 + 5/2 + 3 + 7/2 + \cdots$

   e) $-1 + 1 - 1 + 1 - 1 \cdots$

   f) $3 + 3/2 + 3/4 + 3/8 + 3/16 + \cdots$

   g) $4 + 0 - 4 - 8 - 12 - \cdots$

   h) $3 - 6 + 12 - 24 + 48 - 96 + \cdots$

   i) $1/3 + 1 + 3 + 9 + 27 + 81 + \cdots$

   j) $1 + 3 + 5 + 9 + 17 + 33 + \cdots$

k) $6 + 11 + 16 + 21 + 26 + \cdots$

2. For the geometric series in #1, find the sum S_n.