Geometric Series

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Geometric Series

A geometric series is the sum of the terms in a geometric sequence. Let's consider the geometric sequences and then sums.

$$\sum_1 = 3 + 6 + 12 + 24 + 48 + 96 + \cdots$$
$$\sum_2 = 2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \cdots$$
$$\sum_3 = 2 - 2 + 2 - 2 + 2 - 2 + \cdots$$

Let's consder the sum of the first n terms of the series. We will let the general temr of the series be denoted by t_i. This general term is the same as the general term for a geometric sequence. So, $t_i = ar^i,\ i = 0, 1, 2, ldots$. Now the sum of the first n terms is,

$$\begin{aligned} S_n &= t_0 + t_1 + \cdots + t_n \\ &= a + nar + ar^2 + \cdots + ar^n \\ &= a(1 + r + r^2 + \cdots + r^n) \\ &= a\left(\frac{1 - r^n}{1 - r}\right) \end{aligned}$$

Note: $1 + r + r^2 + \cdots + r^n = \frac{1-r^n}{1-r},\ r \neq 1$. So,

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \text{ or} \tag{1}$$
$$S_n = \frac{a - t_n}{1 - r} \tag{2}$$

Exercises

1. Determine which series are arithmetic.

a) $1+2+3+\cdots$

b) $3+5+7+9+11+\cdots$

c) $1+2+4+8+16+\cdots$

d) $1+3/2+2+5/2+3+7/2+\cdots$

e) $-1+1-1+1-1\cdots$

f) $3+3/2+3/4+3/8+3/16+\cdots$

g) $4+0-4-8-12-\cdots$

h) $3-6+12-24+48-96+\cdots$

i) $1/3+1+3+9+27+81+\cdots$

j) $1+3+5+9+17+33+\cdots$

k) $6 + 11 + 16 + 21 + 26 + \cdots$

2. For the geometric series in #1 state the general term, e.g. $t_n = ar^n$.

