Arithmetic Series

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When considering a series, the sum of the entire series in many cases will be infinite. Instead of trying to find the sume of the entire series, why not consider the sum of the first n terms of the series. We'll denote this sum of the first n terms by S_n. If we let a general term of our series be denotes by t_i, S_n is given by the following sum,

$$S_n = t_1 + t_2 + t_3 + \cdots + t_n$$

Now, what doe t_i look like? Since we are considering an arithmetic series, the general term t_i is equal to the general term of an arithemtic sequence. So,

$$t_i = a + id,\ \ i = 0, 1, 2, \ldots$$

Now, the sume of the first n terms is,

$$\begin{aligned}
S_n &= t_0 + t_1 + \cdots + t_n \\
&= a + (a + d) + (a + 2d) + \cdots + (a + nd) \\
&= a(n + 1) + (1 + 2 + \cdots + n)d \\
&= a(n + 1) + \frac{(n + 1)n}{2}d \\
&= \frac{2a(n + 1) + dn(n + 1)}{2} \\
S_n &= \frac{(2a + nd)(n + 1)}{2},\ \ n = 0, 1, 2, \ldots \\
&= \frac{(a + (a + nd))(n + 1)}{2} \\
&= \frac{(a + an)(n + 1)}{2}
\end{aligned}$$

Note: $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

Therefore,

$$S_n = \frac{(2a+nd)(n+1)}{2} \text{ or} \tag{1}$$

$$S_n = \left(\frac{a+t_n}{2}\right)(n+1) \tag{2}$$

Exercises

1. Determine which series are arithmetic.

    a) $1 + 2 + 3 + \cdots$

    b) $3 + 5 + 7 + 9 + 11 + \cdots$

    c) $1 + 2 + 4 + 8 + 16 + \cdots$

    d) $1 + 3/2 + 2 + 5/2 + 3 + 7/2 + \cdots$

    e) $-1 + 1 - 1 + 1 - 1 \cdots$

    f) $3 + 3/2 + 3/4 + 3/8 + 3/16 + \cdots$

    g) $4 + 0 - 4 - 8 - 12 - \cdots$

    h) $3 - 6 + 12 - 24 + 48 - 96 + \cdots$

    i) $1/3 + 1 + 3 + 9 + 27 + 81 + \cdots$

    j) $1 + 3 + 5 + 9 + 17 + 33 + \cdots$

    k) $6 + 11 + 16 + 21 + 26 + \cdots$

2. For the arithmetic series in # 1 find the sum S_n.