

Arithmetic Series

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2021

Arithmetic Series

When considering a series, the sum of the entire series in many cases will be infinite. Instead of trying to find the sum of the entire series, why not consider the sum of the first n terms of the series. We'll denote this sum of the first n terms by S_n . If we let a general term of our series be denoted by t_i , S_n is given by the following sum,

$$S_n = t_1 + t_2 + t_3 + \cdots + t_n$$

Now, what do t_i look like? Since we are considering an arithmetic series, the general term t_i is equal to the general term of an arithmetic sequence. So,

$$t_i = a + id, \quad i = 0, 1, 2, \dots$$

Now, the sum of the first n terms is,

$$\begin{aligned} S_n &= t_0 + t_1 + \cdots + t_n \\ &= a + (a + d) + (a + 2d) + \cdots + (a + nd) \\ &= a(n + 1) + (1 + 2 + \cdots + n)d \\ &= a(n + 1) + \frac{(n + 1)n}{2}d \\ &= \frac{2a(n + 1) + dn(n + 1)}{2} \\ S_n &= \frac{(2a + nd)(n + 1)}{2}, \quad n = 0, 1, 2, \dots \\ &= \frac{(a + (a + nd))(n + 1)}{2} \\ &= \frac{(a + an)(n + 1)}{2} \end{aligned}$$

Note: $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

Therefore,

$$S_n = \frac{(2a + nd)(n + 1)}{2} \text{ or} \quad (1)$$

$$S_n = \left(\frac{a + t_n}{2}\right)(n + 1) \quad (2)$$

Exercises

1. Determine which series are arithmetic.

a) $1 + 2 + 3 + \dots$

b) $3 + 5 + 7 + 9 + 11 + \dots$

c) $1 + 2 + 4 + 8 + 16 + \dots$

d) $1 + 3/2 + 2 + 5/2 + 3 + 7/2 + \dots$

e) $-1 + 1 - 1 + 1 - 1 \dots$

f) $3 + 3/2 + 3/4 + 3/8 + 3/16 + \dots$

g) $4 + 0 - 4 - 8 - 12 - \dots$

h) $3 - 6 + 12 - 24 + 48 - 96 + \dots$

i) $1/3 + 1 + 3 + 9 + 27 + 81 + \dots$

j) $1 + 3 + 5 + 9 + 17 + 33 + \dots$

k) $6 + 11 + 16 + 21 + 26 + \dots$

2. For the arithmetic series in # 1, state the general term. e.g.
 $t_n = a + nd$.