Geometric Sequences

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Sequences

There are two types of sequences we will consider, *arithmetic* and *geometric*. Regardless of the type of sequence, we will call the general term of the sequence t_n and the first term, $t_1 = a$. The value n is the *position* of the term in the sequence. When writing the term of a sequence, there are two ways: A formula for the *general term* t_n in terms of n and a *recursive* formula for t_n that involves the previous, t_{n-1} term. Let's start with arithmetic sequences.

Geometric sequences

A *geometric sequence* can be thought of a sequence of numbers where the next number, or term, in the sequence, is the previous value or term multiplied by a fixed value, r, say. Let's consider a few examples to see this idea explicitly.

$$\begin{aligned} S_1 &= 3, 6, 12, 24, 48, 96, \ldots \\ S_2 &= 2, 1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, \ldots \\ S_3 &= 2, -2, 2, -2, 2, -2, 2, -2 \ldots \end{aligned}$$

If we look at the three geometric sequences above in greater detail we see that for,
sequence S_1, the sequence starts at 3 and each term is multiplied by 2;
sequecne S_2, the sequence starts at 2 and each term is multiplied by $1/2$;
sequence S_3, the sequence starts at 2 and each term is multiplied by -1.
We call the value that the sequence starts at a and the factor that the sequence changes by r. The *general term* for a geometric sequence is given by,

$$t_n = ar^{n-1}$$

The *recursive* formula for a geometric sequence is given by,

$$t_n = t_{n-1}r$$

Exercises

1. Which sequences are geometric sequences?

a) $\{-18, -7, 4, 15, 26, \ldots\}$

b) $\{1, 2, 4, 8, 16, 32, \ldots\}$

c) $\{1, -1, 1, -1, 1, -1, \ldots\}$

d) $\{-1, 3, 7, 11, 15, \ldots\}$

e) $\{7, 4, 1, -2, -5, \ldots\}$

f) $\{1/2, 1/4, 1/8, 1/16, \ldots\}$

g) $\{5, 7, 9, 11, 13, \ldots\}$

h) $\{3, 15, 75, 375, 1875, \ldots\}$

i) $\{1, 3, 5, 7, 9, \ldots\}$

j) $\{2, 2/3, 2/9, 2/27, 2/81, \ldots\}$

k) $\{20, 200, 2000, 20000, \ldots\}$

l) $\{1, 4, 7, 10, 13, 16, 19, 22, 25, \ldots\}$

n) $\{88, 78, 68, 58, 48, \ldots\}$

o) $\{3, 6, 12, 24, \ldots\}$

m) $\{2/3, 2/9, 2/17, 2/81, \ldots\}$

p) $\{3, 8, 13, 23, 28, 33, 38, \ldots\}$

2. For the geometric sequences in #1 find a, r, the general term and the recursive term for the sequence.