Arithmetic Sequences

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2021

Sequences

There are two types of sequences we will consider, *arithmetic* and *geometric*. Regardless of the type of sequence, we will call the general term of the sequence t_n and the first term, $t_1 = a$. The value n is the *position* of the term in the sequence. When writing the term of a sequence, there are two ways: A formula for the *general term t_n* in terms of n and a *recursive* formula for t_n that involves the previous, t_{n-1} term. Let's start with arithmetic sequences.

Arithmetic sequences

An **arithemic sequence** can be thought of a sequence of numbers where the next number, or term, in the sequence, is the previous value or term plus a fixed value, d, say. Let's consider a few examples to see this idea explicitly.

$$
\begin{aligned}
S_1 &= 1, 3, 5, 7, 9, 11, 13, 15, \ldots \\
S_2 &= 2, 7, 12, 17, 22, 27, 32, 37, \ldots \\
S_3 &= 6, 3, 0, -3, -6, -9, -12, -15, \ldots
\end{aligned}
$$

If we look at the three arithmetic sequences above in greater detail we see that for,
sequence S_1, the sequence starts at 1 and each term is increasing by 2;
sequecne S_2, the sequence starts at 2 and each term is increasing by 5;
sequence S_3, the sequence starts at 6 and each term is decreasing by -3.
We call the value that the sequence starts at a and the amount that the sequence changes by d. The *general term* for an arithmetic sequence is given by,

$$
t_n = a + (n - 1)d
$$

The *recursive* formula for an arithmetic sequence is given by,

$$t_n = t_{n-1} + d$$

Exercises

1. Which sequences are arithemtic sequences?

   a) $\{-18, -7, 4, 15, 26, \ldots\}$

   b) $\{1, 2, 4, 8, 16, 32, \ldots\}$

   c) $\{1, -1, 1, -1, 1, -1, \ldots\}$

   d) $\{-1, 3, 7, 11, 15, \ldots\}$

   e) $\{7, 4, 1, -2, -5, \ldots\}$

   f) $\{1/2, 1/4, 1/8, 1/16, \ldots\}$

   g) $\{5, 7, 9, 11, 13, \ldots\}$

   h) $\{3, 15, 75, 375, 1875, \ldots\}$

   i) $\{1, 3, 5, 7, 9, \ldots\}$

   j) $\{2, 2/3, 2/9, 2/27, 2/81, \ldots\}$

   k) $\{20, 200, 2000, 20000, \ldots\}$

l) $\{1, 4, 7, 10, 13, 16, 19, 22, 25, \ldots\}$ n) $\{88, 78, 68, 58, 48, \ldots\}$

o) $\{3, 6, 12, 24, \ldots\}$

m) $\{2/3, 2/9, 2/17, 2/81, \ldots\}$

p) $\{3, 8, 13, 23, 28, 33, 38, \ldots\}$

2. For the arithmetic sequences in #1 find a, d, the general term and the recursive term for the sequence.