

Solving Inequalities

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Solving Inequalities

Solving equalities involves expressions with an “=” sign in it. Solving *inequalities*, involve expressions with an inequality, $<$, $>$, \leq or \geq in them instead of the “=”. Let’s consider an example.

Example

Solve the following inequality,

$$0 \geq x^2 + 7x + 10$$

Solution

1. First we consider the “equality” or equation by replacing the inequality sign, \geq in this case, with an “=” sign. Doing this we get,

$$0 = x^2 + 7x + 10$$

2. Now we follow the same steps as above for solving equations.

$$0 = x^2 + 7x + 10$$

$$0 = (x + 5)(x + 2)$$

$$x + 5 = 0 \text{ or } x + 2 = 0$$

$$x = -5 \text{ or } x = -2$$

3. Let’s re-insert the inequality sign \geq and see what we get.

$$0 \geq x^2 + 7x + 10$$

$$0 \geq (x + 5)(x + 2)$$

When is the product of two numbers less than or equal to 0? Good question. If we think about this, the product of two numbers is

less than 0 when exactly one number is less than 0 and the product of two numbers is equal to 0 when at least one of the numbers is equal to 0. In our case this means,

$$\begin{aligned}x + 5 &\leq 0 \text{ or } x + 2 \leq 0 \\x &\leq -5 \text{ or } x \leq -2\end{aligned}$$

4. The solution of the inequality is given by,

$$\begin{aligned}0 &\geq x^2 + 7x + 10 \\0 &\geq (x + 5)(x + 2)\end{aligned}$$

where $x \leq -5$ or $x \leq -2$, but not both.

Let's try another example.

Example

Solve the following inequality,

$$0 \leq x^2 + 11x - 12$$

Solution First, let's factor the right hand side.

$$\begin{aligned}0 &\leq x^2 + 11x - 12 \\0 &\leq (x + 12)(x - 1)\end{aligned}$$

When is a product of two numbers greater than or equal to 0? When both values are positive, negative or at least one is 0. With this in mind we have the following,

$$\begin{aligned}x + 12 \geq 0 \text{ and } x - 1 \geq 0 \text{ or } x + 12 \leq 0 \text{ and } x - 1 \leq 0 \\ \text{which implies } x \geq -12 \text{ and } x \geq 1 \text{ or } x \leq -12 \text{ or } x \leq 1.\end{aligned}$$

The solution to the inequality is two:
the first solution is,

$$x \geq -12 \text{ and } x \geq 1$$

or the second solution,

$$x \leq -12 \text{ and } x \leq 1.$$

Exercises

Solve the following inequalities.

(a) $x^3 - 3x^2 + 3x - 1 \leq 0$

(b) $x^2 - 8x + 7 \geq 0$

(c) $0 \geq x^2 - a^2$

(d) $0 \leq x^3 + 2x^2 - x - 2$

(e) $5x^2 + 13x + 6 \leq 0$

(f) $9x^2 + 10x + 1 \geq 0$

(g) $-x^2 + 2x + 3 \geq 0$

(h) $4x + 3 \leq -x^2$

(i) $x^2 \geq 1$

(j) $6x^2 \leq 2x + 28$