# Rules of Differentiation 9


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# Rules of differentiation

There are a number of rules when taking the derivative of a function. B

### Constant Function Rule

If $f(x) = K$ then $f'(x) = 0$.

### The Power Rule

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ where $n$ is a real number, $n \in \mathbb{R}$.

### Constant Multiple Rule

If $f(x) = Kg(x)$ where $K$ is a constant then $f'(x) = Kg'(x)$.

### The Sum Rule

If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = f(x) + g(x)$ then $F'(x) = f'(x) + g'(x)$.

### Difference Rule

$$\text{If } F(x) = f(x) - g(x) \text{ then } F'(x) = f'(x) - g'(x)$$

### The Product Rule

$$\text{If } F(x) = f(x)g(x) \text{ then } F'(x) = f'(x)g(x) + f(x)g'(x)$$

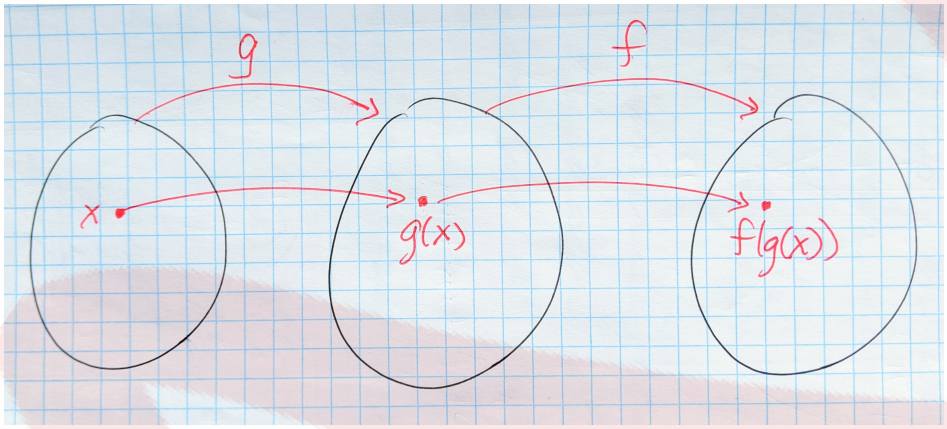
### Power Rule

$$\text{If } f(x) = [g(x)]^n \text{ then } f'(x) = n[g(x)]^{n-1}g'(x), \text{ where } n \in \mathbb{Z}, \ n \text{ is an integer.}$$

### The quotient Rule

$$\text{If } F(x) = \frac{f(x)}{g(x)} \text{ then } F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Before we look at the chain rule for differentiation let's' look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or




The domain of one function is the range of the other function. Given two functions $f$ and $g$ the composite function $f \circ g$ is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

### The Chain Rule

If $f$ and $g$ are differentiable functions and $F(x) = f \circ g(x)$ then

$$\boxed{F'(x) = f'(g(x))g'(x)}$$

Using Leibniz Notation, If $y = f(u),\ u = g(x)$ then,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

**Note:** The power function rule is a special case of the chain rule where $f(x) = x^n$ and given some function $g(x)$, the derivative of $F(x) = f \circ g(x) = [g(x)]^n$ is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

## Exercises

1. Use the Chain Rule to find $\frac{dy}{dx}$ at the given value of $x$.

   a) $y = 3u^2 - 5u + 2,\ u = x^2 - 1,\ x = 2$

   b) $y = 2u^3 + u^2,\ u = x + x^{1/2},\ x = 1$

   c) $y\frac{u^3}{u+1},\ u = (x^2 + 1)^3,\ x = 1$

   d) $y = \frac{1}{(1+u^2)^2},\ u = \sqrt{x} - 1,\ x = 4$

   e) $y = u^5 + u^3,\ u = \frac{3}{\nu} - 4\nu,\ \nu = 3 - x^2,\ x = 2.$

2. If $h(x) = x(2x + 7)^4(x - 1)^2$, find $h'(-3)$.

3. Differentiate,

$$y = \frac{x^2(1-2)^3}{(1+x)^4}$$