Rules of Differentiation 8

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Rules of differentiation

There are a number of rules when taking the derivative of a function. B

Constant Function Rule

If $f(x) = K$ then $f'(x) = 0$.

The Power Rule

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ where n is a real number, $n \in \mathbb{R}$.

Constant Multiple Rule

If $f(x) = Kg(x)$ where K is a constant then $f'(x) = Kg'(x)$.

The Sum Rule

If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = f(x) + g(x)$ then $F'(x) = f'(x) + g'(x)$.

Difference Rule

$$\text{If } F(x) = f(x) - g(x) \text{ then } F'(x) = f'(x) - g'(x)$$

The Product Rule

$$\text{If } F(x) = f(x)g(x) \text{ then } F'(x) = f'(x)g(x) + f(x)g'(x)$$

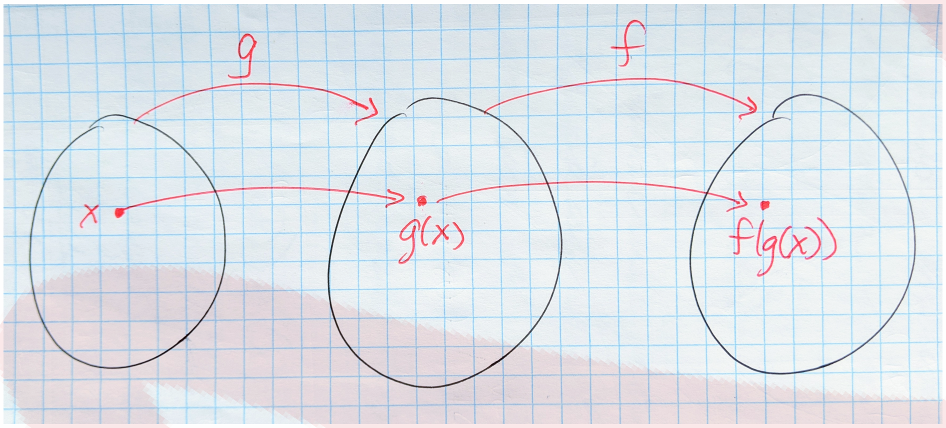
Power Rule

$$\text{If } f(x) = [g(x)]^n \text{ then } f'(x) = n[g(x)]^{n-1}g'(x), \text{ where } n \in \mathbb{Z},\ n \text{ is an integer.}$$

The quotient Rule

$$\text{If } F(x) = \frac{f(x)}{g(x)} \text{ then } F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Before we look at the chain rule for differentiation let's' look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or



The domain of one function is the range of the other function. Given two functions f and g the composite function $f \circ g$ is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

The Chain Rule

If f and g are differentiable functions and $F(x) = f \circ g(x)$ then

$$\boxed{F'(x) = f'(g(x))g'(x)}$$

Using Leibniz Notation, If $y = f(u),\ u = g(x)$ then,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

Note: The power function rule is a special case of the chain rule where $f(x) = x^n$ and given some function $g(x)$, the derivative of $F(x) = f \circ g(x) = [g(x)]^n$ is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

Exercises

1. Differentiate.

a) $g(x) = (2x-1)^4(2-3x)^4$

b)
$$y = \frac{(2x-1)^2}{(x-2)^3}$$

c) $y = x^4(1-4x^2)^3$

d)
$$y = \frac{3x+5}{1-x^2}$$

e)
$$h(x) = \frac{\sqrt{1-x^2}}{1-x}$$

f)
$$s(t) = \left(\frac{t-\pi}{t+6\pi}\right)^{1/3}$$

2. Given $y = f(x^2+3x-5)$ find $\frac{dy}{dx}$ when $x = 1$, given $f'(-1) = 2$.

3. Let $y = g(h(x))$ where $h(x) = \frac{x^2}{x+2}$. If $g'(9/5) = -2$, find $\frac{dy}{dx}$ when $x = 3$.