Rules of Differentiation 5

Raise My MArks

Rules of differentiation

There are a number of rules when taking the derivative of a function. B

Constant Function Rule

If $f(x) = K$ then $f'(x) = 0$.

The Power Rule

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ where n is a real number, $n \in \mathbb{R}$.

Constant Multiple Rule

If $f(x) = Kg(x)$ where K is a constant then $f'(x) = Kg'(x)$.

The Sum Rule

If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = f(x) + g(x)$ then $F'(x) = f'(x) + g'(x)$.

Difference Rule

$$\text{If } F(x) = f(x) - g(x) \text{ then } F'(x) = f'(x) - g'(x)$$

The Product Rule

$$\text{If } F(x) = f(x)g(x) \text{ then } F'(x) = f'(x)g(x) + f(x)g'(x)$$

Power Rule

$$\text{If } f(x) = [g(x)]^n \text{ then } f'(x) = n[g(x)]^{n-1}g'(x), \text{ where } n \in \mathbb{Z}, \ n \text{ is an integer.}$$

The quotient Rule

$$\text{If } F(x) = \frac{f(x)}{g(x)} \text{ then } F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Before we look at the chain rule for differentiation let's' look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or
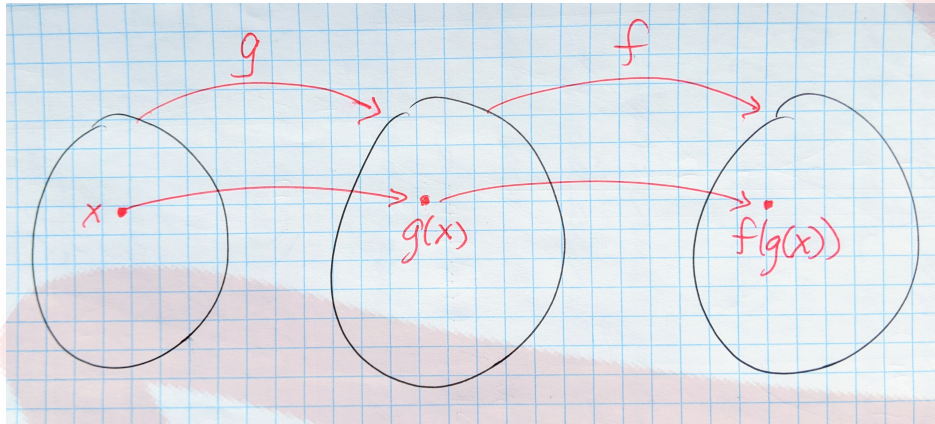
The domain of one function is the range of the other function. Given two functions f and g the composite function $f \circ g$ is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

The Chain Rule

If f and g are differentiable functions and $F(x) = f \circ g(x)$ then

$$\boxed{F'(x) = f'(g(x))g'(x)}$$

Using Leibniz Notation, If $y = f(u), \ u = g(x)$ then,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

Note: The power function rule is a special case of the chain rule where $f(x) = x^n$ and given some function $g(x)$, the derivative of $F(x) = f \circ g(x) = [g(x)]^n$ is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

Exercises

1. Given $f(x) = 3x + 1$, $g(x) = x^3$, $h(x) = \frac{1}{x+1}$ and $u(x) = \sqrt{x}$ find the following functions,

   a) $f \circ u$

   d) $h \circ u$

   b) $g \circ f$

   e) $(f \circ g) \circ u$

   c) $u \circ g$

   f) $g \circ (h \circ u)$

2. Given the following functions $h(x) = f(g(x))$, determine f and g.

   a) $h(x) = (2x^2 - 1)^4$

   c) $h(x) = (x+1)^2 - 9(x+1)$

   b) $h(x) = \frac{1}{x-4}$

   d) $h(x) = \sqrt{2 - x^3}$

e) $h(x) = (x - 2)^2$

3. Find the derivative of the functions in # 2.