

Rules of Differentiation 4

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Rules of differentiation

There are a number of rules when taking the derivative of a function. B

Constant Function Rule

If $f(x) = K$ then $f'(x) = 0$.

The Power Rule

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ where n is a real number, $n \in \mathbb{R}$.

Constant Multiple Rule

If $f(x) = Kg(x)$ where K is a constant then $f'(x) = Kg'(x)$.

The Sum Rule

If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = f(x) + g(x)$ then $F'(x) = f'(x) + g'(x)$.

Difference Rule

If $F(x) = f(x) - g(x)$ then $F'(x) = f'(x) - g'(x)$

The Product Rule

If $F(x) = f(x)g(x)$ then $F'(x) = f'(x)g(x) + f(x)g'(x)$

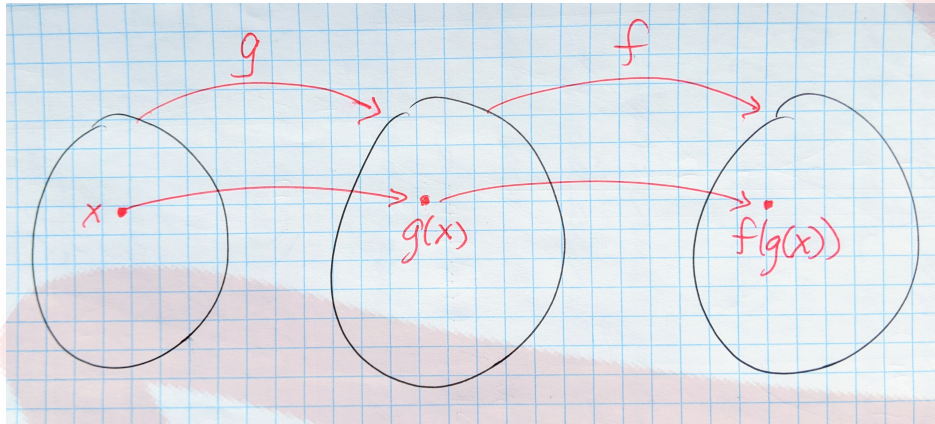
Power Rule

If $f(x) = [g(x)]^n$ then $f'(x) = n[g(x)]^{n-1}g'(x)$, where $n \in \mathbb{Z}$, n is an integer.

The quotient Rule

If $F(x) = \frac{f(x)}{g(x)}$ then $F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Before we look at the chain rule for differentiation let's look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or



The domain of one function is the range of the other function. Given two functions f and g the composite function $f \circ g$ is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

The Chain Rule

If f and g are differentiable functions and $F(x) = f \circ g(x)$ then

$$F'(x) = f'(g(x))g'(x)$$

Using Leibniz Notation, If $y = f(u)$, $u = g(x)$ then,

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Note: The power function rule is a special case of the chain rule where $f(x) = x^n$ and given some function $g(x)$, the derivative of $F(x) = f \circ g(x) = [g(x)]^n$ is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

Exercises

1. Keep Differentiating.

a)

$$y = \frac{x^2 - 1}{x(3x + 1)}$$

c)

$$h(t) = \frac{2t - 3}{t + 5}$$

b)

$$h(x) = \frac{x^3}{2x^2 - 1}$$

d)

$$h(t) = \frac{t}{t + 1}$$

2. Find the following composite function $f \circ g(x)$ given f and g .

a)

$$f(x) = x^2, \quad g(x) = \sqrt{x}$$

b)

$$f(x) = \frac{1}{x}, \quad g(x) = \sqrt{x + 2}$$