Rules of Differentiation 2

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2020

Rules of differentiation

There are a number of rules when taking the derivative of a function. B

Constant Function Rule

If $f(x) = K$ then $f'(x) = 0$.

The Power Rule

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ where n is a real number, $n \in \mathbb{R}$.

Constant Multiple Rule

If $f(x) = Kg(x)$ where K is a constant then $f'(x) = Kg'(x)$.

The Sum Rule

If $f(x)$ and $g(x)$ are differentiable functions and $F(x) = f(x) + g(x)$ then $F'(x) = f'(x) + g'(x)$.

Difference Rule

$$\text{If } F(x) = f(x) - g(x) \text{ then } F'(x) = f'(x) - g'(x)$$

The Product Rule

$$\text{If } F(x) = f(x)g(x) \text{ then } F'(x) = f'(x)g(x) + f(x)g'(x)$$

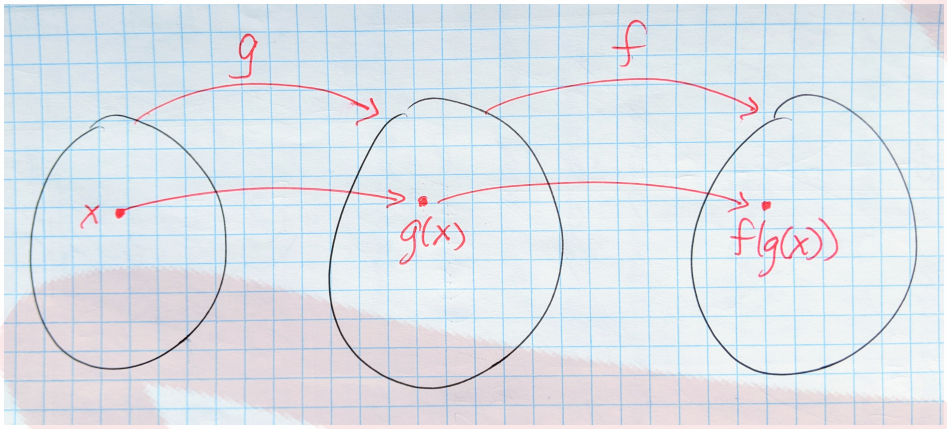
Power Rule

$$\text{If } f(x) = [g(x)]^n \text{ then } f'(x) = n[g(x)]^{n-1}g'(x), \text{ where } n \in \mathbb{Z}, \ n \text{ is an integer.}$$

The quotient Rule

$$\text{If } F(x) = \frac{f(x)}{g(x)} \text{ then } F'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Before we look at the chain rule for differentiation let's' look at the composition of functions. The *composition of two or more functions* can be thought of as taking the function of a function or



The domain of one function is the range of the other function. Given two functions f and g the composite function $f \circ g$ is defined by,

$$(f \circ g)(x) = f(g(x))$$

The **chain rule** considers the derivative of the composition of two functions.

The Chain Rule

If f and g are differentiable functions and $F(x) = f \circ g(x)$ then

$$\boxed{F'(x) = f'(g(x))g'(x)}$$

Using Leibniz Notation, If $y = f(u),\ u = g(x)$ then,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

Note: The power function rule is a special case of the chain rule where $f(x) = x^n$ and given some function $g(x)$, the derivative of $F(x) = f \circ g(x) = [g(x)]^n$ is

$$f'(x) = n[g(x)]^{n-1}g'(x).$$

Exercises

Use the rules of differentiation to differentiate the following functions.

a) $f(x) = (x^2 + 3x + 5)^6$

b) $f(x) = 2x(x + 1)^3(x^2 + 2x + 1)^2$

c) $y = x^2(3x^2 + 4)^2(3 - x^3)^4$

d) $y = 2(x - 29)(x + 1)$

e) $y = (x^3 - 5x + 2)(3x^2 - 2x)$

f) $y = 3x(x - 4)(x + 3)$

g) $y = (2x + 1)^5(3x + 2)^4$

h) $y = (4x^2 + 2x)(3 - 2x - 5x^2)$

i) $y = (1 - 2x)(1 + 2x)$

j) $y = (1 - x^2)^4(2x + 6)^3$

k) $y = (5x + 1)^3(2x + 6)^3$

l) $y = (5x + 1)^3(x - 4)$

n) $h(x) = (5x^7 + 1)(x^2 - 2x)$

m) $s(t) = (t^2 + 1)(3 - 2t^2)$

o) $h(x) = x^2(2x - 1)$