# Remainder Theorem

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## Remainder Theorem

The remainder theorm gives a relationship between the dividend $f(x)$ and the remainder, $r(x)$.

## Example

$f(x) = x^3 - x^2 - 4x - 2$ divided by $d(x) = x + 2$. First let's consider, $d(x) = x + 2 = 0$ and solve for $x$,

**Solutions:**

$$d(x) = x + 2 = 0 \implies x = -2$$

Now, let's consider $f(-2)$.

$$\begin{aligned} f(-2) &= (-2)^3 - (-2)^2 - 4(-2) - 2 \\ &= -8 - 4 + 8 - 2 \\ &= -6 \end{aligned}$$

Now let's divide $f(x) \div d(x)$ and see what we get.

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x + 2 & x^3 - x^2 - 4x - 2 \\ & -(x^3 + 2x^2) \\ \cline{2-2} & \quad -3x^2 - 4x \\ & \quad -(-3x^2 - 6x) \\ \cline{2-2} & \qquad 2x - 2 \\ & \qquad -(2x + 4) \\ \cline{2-2} & \qquad\quad -6 \end{array}$$

We have $q(x) = x^2 - 3x + 2$ and $r(x) = -6 = f(-2)$.

## Example

Let $f(x) = x^3 - 4x^2 + 5x - 1$ be divided by $d(x) = x - 2$.

**Solutions:**

$$
\begin{array}{r|l}
 & x^2 - 2x + 1 \\
\hline
x-2 & x^3 - 4x^2 + 5x - 1 \\
 & -(x^3 - 2x^2) \\
\hline
 & \quad -2x^2 + 5x \\
 & \quad -(-2x^2 + 4x) \\
\hline
 & \qquad -x - 1 \\
 & \qquad -(-x-2) \\
\hline
 & \qquad\qquad 1
\end{array}
$$

and $d(x) = x - 2 = 0$ gives, $x = 2$,

$$f(2) = 2^3 - 4(2)^2 + 5(2) - 1 = 8 - 16 + 10 - 1 = 1 = r(x).$$

## Remainder Theorem

What does the Remainder Theorem say?

If $f(x)$ is divided by $x - p$, giving a quotient $q(x)$ and a remainder $r$ then $r = f(p)$.

### Example

Find the remainder when $f(x) = x^3 - 4x^2 + 5x - 1$ is divided by $2x - 3$. Let's rewrite $2x - 3$ in the form $x - p$,

$$2x - 3 = 2\left(x - \frac{3}{2}\right).$$

**Solutions:** Then by the remainder theorem,

$$
\begin{aligned}
f(x) &= f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 - 4\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) - 1 \\
&= \frac{27}{8} - \frac{36}{4} + \frac{15}{2} - 1 \\
&= \frac{27}{8} - \frac{72}{8} + \frac{60}{8} - \frac{8}{8} \\
&= \frac{7}{8} \\
&= r
\end{aligned}
$$

### Example

What is the remainder when, $x^3 - 4x^2 + 2x - 6$ is divided by $x + 1$?

**Solutions:** So,

$$d(x) = x + 1 = 0 \implies x = -1 \text{ so, } p = -1.$$

$$\begin{aligned} r &= f(p) = f(-1) = (-1)^3 - 4(-1)^2 + 2(-1) - 6 \\ &= -1 - 4 - 2 - 6 \\ &= -13 \end{aligned}$$

Therefore, the remainder is $-13$.

## Exercises

1. Use the Remainder Theorm to find the remainder of the following:

   a) $(x^3 - 4x^2 + 2x + 6) \div (2x + 3)$

   b) $(3x^5 - 5x^2 + 4x + 1) \div (2x - 1)$

   c) $(4x^3 + 9x - 10) \div (x - 1)$

   d) $(6x^2 - 10x + 7) \div (3x + 1)$

   e) $(x^4 - x^3 + x^2 - 3x + 4) \div (x - 5)$

2. Perform the following,

   a) $(x^4 - 4x^3 + 3x^2 - 3) \div (x^2 + x - 2)$

   b) $(x^3 + 2x^2 - x - 2) \div (x - 1)$

   c) $(3x^3 + x + 2) \div (3x - 1)$

   d) $(6x^3 + 31x^2 + 25x - 12) \div (2x + 3)$

   e) $(4x^4 + 8x^3 - x^2 + x + 3) \div (x - 5)$