

Properties of Logarithms 4

**Raise My**  
**MA** **rks**

[RaiseMyMarks.com](https://www.RaiseMyMarks.com)

2020

## Properties of logarithm

Let's consider some properties of the logarithmic function,

$$y = f(x) = \log_b x \quad (1)$$

1.  $b > 0$
2.  $x$ -intercept = 1
3.  $y$ -axis is a vertical asymptote
4. Domain =  $\{x \in \mathbb{R} | x > 0\}$
5. Range =  $\{y | y \in \mathbb{R}\}$
6. If  $b > 1$  then the logarithmic function is increasing.
7. If  $0 < b < 1$  then the logarithm function is decreasing.

Notes, the most common base used is 10 for the logarithm function. This logarithm is written as  $\log x$  rather than  $\log_{10} x$ . The value of the base  $b$  can be omitted when  $b = 10$ .

### Some basic properties of logarithms

1.  $\log_b 1 = 0$
2.  $\log_b b = 1$
3.  $\log_b b^x = x$
4.  $b^{\log_b x} = x$

More properties of logarithms when  $x > 0$ ,  $w > 0$  and  $r \in \mathbb{R}$  is a real number.

5.

$$\log_a(xw) = \log_a x + \log_a w$$

6.

$$\log_a \left( \frac{x}{w} \right) = \log_a x - \log_a w$$

7.

$$\log_a x^r = r \log_a x$$

Let's use some of these properties to solve logarithmic equation.

**Example**

Solve  $\log_6 x = 2$ .

**Solution:**  $\log_6 x = 2$  means  $x = 6^2 = 36$ .

**Example**

Solve  $\log_6 x + \log_6(x + 1) = 1$

**Solution:**

$$\begin{aligned} \log_6 x + \log(x + 1) &= 1, \quad \text{multiplicative property} \\ \log_6[(x(x + 1))] &= 1 \quad \text{equivalence to exponential} \\ 6^1 &= x(x + 1) \\ 0 &= x^2 + x - 6 \\ 0 &= (x + 3)(x - 2) \end{aligned}$$

Therefore,  $x = -3$  or  $2$ .

**Example**

Solve  $3^x = 23$ .

**Solution:**

$$\begin{aligned} 3^x &= 23, \quad \text{Take log base 10 on both sides} \\ \log 3^x &= \log 23, \quad \text{power property} \\ x \log 3 &= \log 23, \quad \text{solve for x} \\ x &= \frac{\log 23}{\log 3} \end{aligned}$$

## Exercises

Evaluate.

a)  $\log_3 135 - \log_3 5$

f)  $\log_5 5^3$

b)  $\log_8 640 - \log_8 10$

g)  $\log_4 41 + \log_3 3^3$

c)  $\log_2 224 - \log_2 7$

h)  $\log_2 14 + \log_2 \left(\frac{4}{7}\right)$

d)  $\log_3 3 + \log_5 1$

i)  $\log_5 200 - \log_5 8$

e)  $\log_4 16 - \log_4 1$

j)  $\log_6 4 + \log_6 54$