Properties of Logarithms

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Properties of logarithm

Let's consider some properties of the logarithmic function,

$$y = f(x) = \log_b x \tag{1}$$

1. $b > 0$

2. $x-$intercept $= 1$

3. $y-$axis is a vertical asymptote

4. Domain$= \{x \in \mathbb{R} | x > 0\}$

5. Range $= \{y | y \in \mathbb{R}\}$

6. If $b > 1$ then the logarithmic function is increasing.

7. If $0 < b < 1$ then the logarithm function is decreasing.

Notes, the most common base used is 10 for the logarithm function. This logarithm is writtedn as $\log x$ rather than $\log_{10} x$. The value of the base b can be omitted when $b = 10$.

Some basic properties of logarithms

1. $\log_b 1 = 0$

2. $\log_b b = 1$

3. $\log_b b^x = x$

4. $b^{\log_b x} = x$

More properties of logarithms when $x > 0, w > 0$ and $r \in \mathbb{R}$ is a real number.

5.
$$\log_a(xw) = \log_a x + \log_a w$$

6.
$$\log_a \left(\frac{x}{w} \right) = \log_a x - \log_a w$$

7.
$$\log_a x^r = r \log_a x$$

Let's use some of these properties to solve logarithmic equation.

Example

Solve $\log_6 x = 2$.

Solution: $\log_6 x = 2$ means $x = 6^2 = 36$.

Example

Solve $\log_6 x + \log_6(x+1) = 1$

Solution:

$$
\begin{aligned}
\log_6 x + \log(x+1) &= 1, \quad \text{multiplicative property} \\
\log_6[(x(x+1))] &= 1 \quad \text{equivalence to exponential} \\
6^1 &= x(x+1) \\
0 &= x^2 + x - 6 \\
0 &= (x+3)(x-2)
\end{aligned}
$$

Therefore, $x = -3$ or 2.

Example

Solve $3^x = 23$.

Solution:

$$
\begin{aligned}
3^x &= 23, \quad \text{Take log base 10 on both sides} \\
\log 3^x &= \log 23, \quad \text{power property} \\
x \log 3 &= \log 23, \quad \text{solve for x} \\
x &= \frac{\log 23}{\log 3}
\end{aligned}
$$

Exercises

Evaluate.

a) $\log_2 8$

f) $\log_2 16^{1/3}$

b) $\log_3 \left(\frac{1}{27}\right)$

g) $\log_6 \sqrt{36} - \log_{25} 5$

c) $\log_2 4^2$

h) $] \log_7 49 + \log_2 64$

d) $\log_5 \sqrt{5}$

i) $log_9 \left(\frac{1}{3}\right) + \log_3 \left(\frac{1}{9}\right)$

e) $\log_3(9 \times \sqrt[5]{9})$

j) $\log_2 \sqrt[4]{32}$