Motion on a straight line

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An application of higher order derivatives is velocity and acceleration calculations. We start with the function $s(t)$ representing the *position* of an object at time t. the first derivative of the position function is called the *velocity* of the object at that time t, $\nu(t) = s'(t)$. The absolute value of the velocity is called the *speed* of the object. $|\nu| = |\nu(t)|$. the second derivative of the position or first derivative of the velocity is the *acceleration* of the object, $a(t) = \nu'(t) = s''(t)$. Let's consider an example.

Example

Let $s(t) = 6t^2 - t^3,\ t \geq 0$ be the position of an object in metres at time t seconds.

a) Find the veolcity at $t = 3$ seconds.

b) When is the object at rest?

c) What is the acceleration at $t = 3$?

d) Which direction is the object travlling in at $t = 4$ seconds?

Solution:

a) We need to take the first derivative of the position $s(t)$ to find the velocity of the object.

$$\nu(t) = s'(t) = 12t - 3t^2$$

is the veolcity at t seconds. When $t = 3$,

$$\nu(3) = 12(3) - 3(3)^2 = 36 - 3(9) = 36 - 27 = 9$$

The veolcity is $9m/s$ at 3 seconds.

b) The object is at rest when it is not moving or when $\nu(t) = 0$. We need to solve $\nu(t) = 0$ for t to determine *when* the object is at rest.

$$\begin{aligned} 12t - 3t^2 &= 0 \\ 3t(4 - t) &= 0 \\ \therefore t &= 0, 4 \end{aligned}$$

Therefore, the object is at rest at $t = 0$ seconds at and $t = 4$ seconds.

c) The acceleration is the second derivative of the position $s(t)$ or the first derivative of the velocity $\nu(t)$.

$$a(t) = \nu'(t) = 12 - 6t$$

is the acceleration at t seconds. When $t = 3$ the acceleration is,

$$a(3) = 12 - 6(3) = 12 - 8 = -6$$

Therefore, the acceleration is $-6m/s^2$ at 3 seconds.

d) To find the direction of the object at $t = 4$ secoonds we need to find the velocity at $t = 4$ seconds.

$$\begin{aligned} \nu(4) &= 12(4\_3(4)^2 \\ &= 48 - 3(16) \\ &= 48 - 48 \\ &= 0 \end{aligned}$$

Since $\nu(4) = 0$, it looks like the object is not moving at 4 seconds.

To summaraize the motion on a straight line: An object that moves along a straight line with position determined by $s(t)$; velocity of $\nu(t) = s'(t)$; acceleration of $a(t) = \nu'(t) = s''(t)$. In Leibniz notation we have,

$$\boxed{\nu = \frac{ds}{dt}, \qquad a = \frac{d\nu}{dt} = \frac{d^2s}{dt^2}}$$

The *speed* of the object is

$$\boxed{\text{speed} = \nu = |\nu(t)|}$$

Exercises

1. Find the veolcity given the position.

a) $s(t) = st^2 - 3t + 15$

f) $s(t) = t - 8 + \frac{6}{t}$

b) $s(t) = \frac{9t}{t+3}$

g) $-\frac{1}{3}t^2 + t + 4 = s(t)$

c) $s(t)2t^3 + 36t - 10$

h) $s(t) = t(t-3)^2$

d) $s(t) = \sqrt{t+1}$

i) $s(t) = t^3 - 7t^2 + 10t$

e) $s(t) = (t-3)^2$

j) $s(t) = t^3 - 12t - 9$

2. Find the acceleration for each object in # 2 at time t.