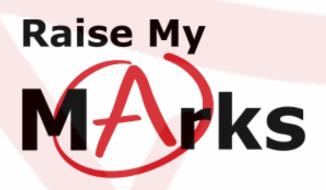
Motion on a straight line



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## Motion on a straight line

An application of higher order derivatives is velocity and acceleration calculations. We start with the function s(t) representing the *position* of an object at time t. the first derivative of the position function is called the *velocity* of the object at that time t,  $\nu(t) = s'(t)$ . The absolute value of the velocity is called the *speed* of the object.  $|\nu| = |\nu(t)|$ . the second derivative of the position or first derivative of the velocity is the *acceleration* of the object,  $a(t) = \nu'(t) = s''(t)$ . Let's consider an example.

## Example

Let  $s(t) = 6t^2 - t^3$ ,  $t \ge 0$  be the position of an object in metres at time t seconds.

- a) Find the veolcity at t = 3 seconds.
- b) When is the object at rest?
- c) What is the acceleration at t = 3?
- d) Which direction is the object traviling in at t = 4 seconds?

## Solution:

a) We need to take the first derivative of the position s(t) to find the velocity of the object.

$$v(t) = s'(t) = 12t - 3t^2$$

is the veolcity at t seconds. When t = 3,

$$\nu(3) = 12(3) - 3(3)^2 = 36 - 3(9) = 36 - 27 = 9$$

The veolcity is 9m/s at 3 seconds.

b) The object is at rest when it is not moving or when  $\nu(t) = 0$ . We need to solve  $\nu(t) = 0$  for t to determine when the object is at rest.

$$12t - 3t^2 = 0$$
  

$$3t(4 - t) = 0$$
  

$$\therefore t = 0, 4$$

Therefore, the object is at rest at t = 0 seconds at and t = 4 seconds.



c) The acceleration is the second derivative of the position s(t) or the first derivative of the velocity  $\nu(t)$ .

$$a(t) = \nu'(t) = 12 - 6t$$

is the acceleration at t seconds. When t = 3 the acceleration is,

$$a(3) = 12 - 6(3) = 12 - 8 = -6$$

Therefore, the acceleration is  $-6m/s^2$  at 3 seconds.

d) To find the direction of the object at t = 4 seconds we need to find the velocity at t = 4 seconds.

$$\nu(4) = 12(4_{-}3(4)^{2})$$
  
= 48 - 3(16)  
= 48 - 48  
= 0

Since  $\nu(4) = 0$ , it looks like the object is not moving at 4 seconds.

To summarize the motion on a straight line: An object that moves along a straight line with position determined by s(t); velocity of  $\nu(t) = s'(t)$ ; acceleration of  $a(t) = \nu'(t) = s''(t)$ . In Leibniz notation we have,

$$\nu = \frac{ds}{dt}, \quad a = \frac{d\nu}{dt} = \frac{d^2s}{dt^2}$$

The *speed* of the object is

speed = 
$$\nu = |\nu(t)|$$



## Exercises

1. Find the veolcity given the position.

a) 
$$s(t) = st^2 - 3t + 15$$
  
f)  $s(t) = t - 8 + \frac{6}{t}$ 

b) 
$$s(t) = \frac{9t}{t+3}$$
 g)  $-\frac{1}{3}t^2 + t + 4 = s(t)$ 

c) 
$$s(t)2t^3 + 36t - 10$$
  
h)  $s(t) = t(t-3)^2$ 

d) 
$$s(t) = \sqrt{t+1}$$
 i)  $s(t) = t^3 - 7t^2 + 10t$ 

e) 
$$s(t) = (t-3)^2$$
 j)  $s(t) = t^3 - 12t - 9$ 

2. Find the acceleration for each object in # 2 at time t.