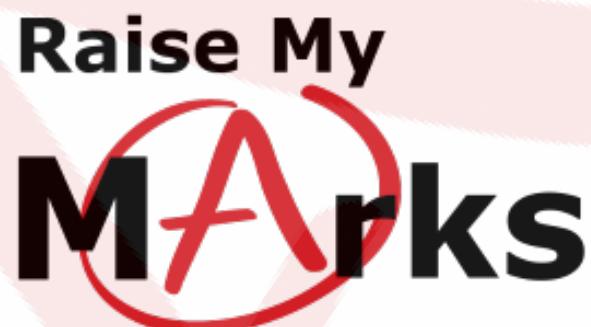


Logarithmic Differentiation 4



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## Logarithmic Differentiation

Let's consider an example.

### Example

For  $y = x^x$ ,  $x > 0$ . Determine  $\frac{dy}{dx}$ .

**Solution:**

$$\begin{aligned}
 y &= x^x, \text{ Take the natural logarithm, ln, of both sides.} \\
 \ln y &= \ln x^x \\
 \ln y &= x \ln x, \text{ Differentiate w.r.t. } x \text{ on both sides.} \\
 \frac{1}{y} \frac{dy}{dx} &= \ln x + \frac{x}{x} \\
 \frac{1}{y} \frac{dy}{dx} &= \ln x + 1 \\
 \frac{dy}{dx} &= y(\ln x + 1) \\
 \therefore \frac{dy}{dx} &= x^x(\ln x + 1)
 \end{aligned}$$

### Example

Differentiate  $y = x^n$ .

**Solution:**

$$\begin{aligned}
 y &= x^n \\
 \ln y &= \ln x^n, \text{ Take the natural logarithm, ln, of both sides.} \\
 \ln y &= n \ln x \\
 \frac{1}{y} \frac{dy}{dx} &= \frac{n}{x} \\
 \frac{dy}{dx} &= \frac{ny}{x} \\
 &= \frac{nx^n}{x} \\
 \therefore \frac{dy}{dx} &= nx^{n-1}
 \end{aligned}$$

**Example**

Differentiate  $y = (x^2 + 3)^x$ .

**Solution:**

$$\begin{aligned}
 y &= (x^2 + 3)^x \\
 \ln y &= \ln(x^2 + 3)^x \\
 \ln y &= x \ln(x^2 + 3) \\
 \frac{1}{y} \frac{dy}{dx} &= \ln(x^2 + 3) = \frac{x(2x)}{x^2 + 3} \\
 &= \ln(x^2 + 3) + \frac{2x^2}{x^2 + 3} \\
 \frac{dy}{dx} &= y \left( \ln(x^2 + 3) + \frac{2x^2}{x^2 + 3} \right) \\
 \frac{dy}{dx} &= (x^2 + 3)^x \left( \ln(x^2 + 3) + \frac{2x^2}{x^2 + 3} \right)
 \end{aligned}$$

**Example**

Differentiate

$$y = \frac{(x^4 + 1)\sqrt{x+2}}{(2x^2 + 2x + 1)}$$

**Solution:** We'll use logarithmic differentiation.

$$\begin{aligned}
 y &= \frac{(x^4 + 1)\sqrt{x+2}}{(2x^2 + 2x + 1)} \\
 \ln y &= \ln \left( \frac{(x^4 + 1)\sqrt{x+2}}{(2x^2 + 2x + 1)} \right) \\
 \ln y &= \ln(x^4 + 1) + \ln \sqrt{x+2} - \ln(2x^2 + 2x + 1) \\
 \frac{1}{y} \frac{dy}{dx} &= \frac{4x^3}{x^4 + 1} + \frac{1}{2} \frac{1}{x+2} - \frac{4x+2}{2x^2 + 2x + 1} \\
 \frac{dy}{dx} &= y \left( \frac{4x^3}{x^4 + 1} + \frac{1}{2} \frac{1}{x+2} - \frac{4x+2}{2x^2 + 2x + 1} \right) \\
 &= \frac{(x^4 + 1)\sqrt{x+2}}{(2x^2 + 2x + 1)} \left( \frac{4x^3}{x^4 + 1} + \frac{1}{2} \frac{1}{x+2} - \frac{4x+2}{2x^2 + 2x + 1} \right)
 \end{aligned}$$

## Exercises

Differentiate.

$$\text{a) } y = x^{\sqrt{10}} - 3$$

$$\text{f) } y = x^{\sqrt{x}}$$

$$\text{b) } y = x^e + e^x$$

$$\text{g) } f(x) = x^x$$

$$\text{c) } y = x^{\ln x}$$

$$\text{h) } f(x) = \frac{(x-3)^2 \sqrt[3]{x+1}}{(x-4)^5}$$

$$\text{d) } y = \frac{(x+1)(x-3)^2}{(x+2)^3}$$

$$\text{i) } y = \frac{1}{(x+1)(x+2)(x+3)}$$

$$\text{e) } s = \left(\frac{1}{t}\right)^t$$