

Logarithmic Differentiation 2

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Logarithmic Differentiation

Let's consider an example.

Example

For $y = x^x$, $x > 0$. Determine $\frac{dy}{dx}$.

Solution:

$$y = x^x, \text{ Take the natural logarithm, } \ln, \text{ of both sides.}$$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x, \text{ Differentiate w.r.t. } x \text{ on both sides.}$$

$$\frac{1}{y} \frac{dy}{dx} = \ln x + \frac{x}{x}$$

$$\frac{1}{y} \frac{dy}{dx} = \ln x + 1$$

$$\frac{dy}{dx} = y(\ln x + 1)$$

$$\therefore \frac{dy}{dx} = x^x(\ln x + 1)$$

Example

Differentiate $y = x^n$.

Solution:

$$y = x^n$$

$$\ln y = \ln x^n, \text{ Take the natural logarithm, } \ln, \text{ of both sides.}$$

$$\ln y = n \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{n}{x}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{ny}{x} \\ &= \frac{nx^n}{x} \end{aligned}$$

$$\therefore \frac{dy}{dx} = nx^{n-1}$$

Example

Differentiate $y = (x^2 + 3)^x$.

Solution:

$$\begin{aligned} y &= (x^2 + 3)^x \\ \ln y &= \ln(x^2 + 3)^x \\ \ln y &= x \ln(x^2 + 3) \\ \frac{1}{y} \frac{dy}{dx} &= \ln(x^2 + 3) = \frac{x(2x)}{x^2 + 3} \\ &= \ln(x^2 + 3) + \frac{2x^2}{x^2 + 3} \\ \frac{dy}{dx} &= y \left(\ln(x^2 + 3) + \frac{2x^2}{x^2 + 3} \right) \\ \frac{dy}{dx} &= (x^2 + 3)^x \left(\ln(x^2 + 3) + \frac{2x^2}{x^2 + 3} \right) \end{aligned}$$

Example

Differentiate

$$y = \frac{(x^4 + 1)\sqrt{x + 2}}{(2x^2 + 2x + 1)}$$

Solution: We'll use logarithmic differentiation.

$$\begin{aligned} y &= \frac{(x^4 + 1)\sqrt{x + 2}}{(2x^2 + 2x + 1)} \\ \ln y &= \ln \left(\frac{(x^4 + 1)\sqrt{x + 2}}{(2x^2 + 2x + 1)} \right) \\ \ln y &= \ln(x^4 + 1) + \ln \sqrt{x + 2} - \ln(2x^2 + 2x + 1) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{4x^3}{x^4 + 1} + \frac{1}{2} \frac{1}{x + 2} - \frac{4x + 2}{2x^2 + 2x + 1} \\ \frac{dy}{dx} &= y \left(\frac{4x^3}{x^4 + 1} + \frac{1}{2} \frac{1}{x + 2} - \frac{4x + 2}{2x^2 + 2x + 1} \right) \\ &= \frac{(x^4 + 1)\sqrt{x + 2}}{(2x^2 + 2x + 1)} \left(\frac{4x^3}{x^4 + 1} + \frac{1}{2} \frac{1}{x + 2} - \frac{4x + 2}{2x^2 + 2x + 1} \right) \end{aligned}$$

Exercises

1. Find the derivative of the following. Recall

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

a) $y = \ln |x|$

b) $y = \ln |2x + 1|$

c) $y = x^2 \ln |x|$

2. Find the slope of the tangent,

a) To the curve $xe^y + y \ln x = 2$ at $(1, \ln 2)$.

b) to the curve $\ln \sqrt{xy} = 0$ at the point $(1/3, 3)$.