Limits

Raise My MArks

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Definition of the Derivative

The *difference quotient*

$$\frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{h}$$

is called the average rate of change of y with respect to x over the interval from $x = a$ to $x = a + h$. The *rate of change of* $y = f(x)$ *with respect to* x *when* $x = a$ is given by,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.
We've been using this,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

notation, or limits.

Limit Definition

Let's define a *limit*. A number L is the *limit* of a function $y = f(x)$ as x approaches a value, say a, if,

$$\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

and is written as

$$\lim_{x \to a} f(x) = L$$

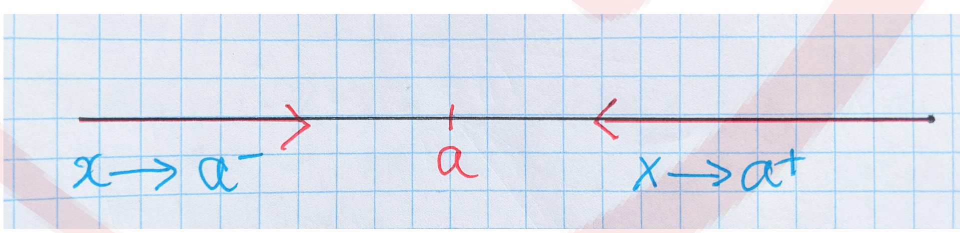
If the above does not hold then the limit does not exists and the limit,

$$\lim_{x \to a} f(x)$$

does not exists.
What does

$$lim_{x \to a^-} f(x)$$

mean? This notation $x \to a^-$ means, as x approaches a from the negative or left hand side of a. And equivalently, $x \to a^+$ means as x approaches a from the positive or right had side of a. Consdering a number line we have,



Properties of Limits

For any real number a suppose functions f and g have limits at $x = a$. Then

1. $$\lim_{x\to a} K = K, \text{ for any constant} K.$$

2. $$\lim_{x\to a} x = a$$

3. $$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

4. $$\lim_{x\to a}[cf(x)] = c\left[\lim_{x\to a} f(x)\right], \text{ for any constant } c$$

5. $$\lim_{x\to a}[f(x)g(x)] = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right]$$

6. $$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ provided } \lim_{x\to a} g(x) \neq 0$$

7. $$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n, \text{ for } n \text{ a rational number.}$$

Exercises

1. Simplify each of the following.

a) $$\frac{(2+h)^2-4}{h}$$

b) $$\frac{\frac{1}{1+h}-1}{h}$$

c) $$\frac{\frac{3}{4+h}-\frac{3}{4}}{h}$$

d) $$\frac{3(1+h)^2-3}{h}$$

e) $$\frac{\frac{-1}{2+h}+\frac{1}{2}}{h}$$

f) $$\frac{(5+h)^3-125}{h}$$

2. Calculate the limits.

a) $\lim_{x\to -4} x$

b) $\lim_{x\to 3}(x-4)$

c) $\lim_{x\to -1} x^3$

d) $\lim_{x\to 15} 7$

e) $\lim_{x\to -1}(4-2x^2)$

f) $\lim_{x\to -3} \frac{|x+3|}{x+3}$

g) $\lim_{x\to -1}(9-x^2)$

h) $\lim_{x\to 0} \sqrt{\frac{x+20}{2x+5}}$

i) $\lim_{x\to 3^+} \frac{1}{x+2}$

k) $\lim_{x\to 0} \sqrt{3+\sqrt{1+x}}$

j) $\lim_{x\to 9} \left(x+\frac{1}{\sqrt{x}}\right)^2$