Polynomial Inequalities

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Polynomial Inequalities

Let's remind ourselves what an inequality is. An "equality" is any expression that involves an equal sign "=". The following are examples of *equalities*.

$$\begin{aligned} 4 &= 2+2 \\ 6+7 &= 13+3 \\ 4x+2 &= x-10 \end{aligned}$$

An "inequality" is an expression that involves an *inequality* sing such as $<$, $>$, $\leq$, $\geq$. For example,

$$\begin{aligned} 3 &< 10 \\ 6-2 &\leq 4 \\ 9 &\geq -2+3 \\ 6x-2 &\geq x+1 \end{aligned}$$

We know how to solve polynomial equations. For example,

$$\begin{aligned} x^2+5x+4 &= 0 \\ (x+4)(x+1) &= 0 \\ \therefore x &= -4, \ -1 \end{aligned}$$

This means, when $x=-4$ or -1 then the equation,

$$x^2+5x+4=0$$

is satisfied. let's consider an pllynomial *inequality*. For example,

$$\begin{aligned} x^2+5x+4 &\leq 0 \\ (x+4)(x+1) &\leq 0 \end{aligned}$$

Steps for solving a polynomial inequality:

1. First, consider the *equality*. This means, $x=-4$ or -1 in order for the equality $(x+4)(x+1)=0$ to be satisfied. But, we're interestd in the *inequality* $(x+4)(x+1)\leq 0$.
2. When is a product less than 0? A product is less than 0 when at least on factor is less than 0 or negative or when an odd number of factors of the product are less than 0. So in our example,

$$(x+4)(x+1)\leq 0$$

when,

$$\begin{aligned} (x+4)(x+1) &\leq 0 \\ \text{when } x+4<0 &\text{ or } x+1<0 \\ \text{or } x<-4 &\text{ or } x<-1. \end{aligned}$$

This means, we need either $x < -4$ **OR** $x < -1$ but **not both**! However, when $x < -4$ then $x < -1$ also holds, so we cannot have $x < -4$. This means, $x \leq -1$ but $x \geq -4$ or we can rewrite this as,

$$-4 \leq x \leq -1.$$

3. Our solution to the polynomial inequality

$$x^2 + 5x + 4 \leq 0$$

is the set,

$$\{x \in \mathbb{R} | -4 \leq x \leq -1\}$$

Exercises

1. Solve each of the following.

   a) $2x^3 + x^2 - 5x + 2 \leq 0$

   b) $-x^2 + 4x - 4 \geq 0$

   c) $x^3 - 10x - 2 \geq 0$

   d) $-x^3 + 9x \geq 0$

   e) $(x + 3)(x - 1) \leq 0$

2. Solve for x.

   a) $|2x - 1| = 7$

   b) $|x + 4| \geq 5$

   c) $|3x + 2| = 6$

   d) $|2x - 3| < 4$

   e) $|x - 3| \leq 9$