Increasing and Decreasing Functions 3

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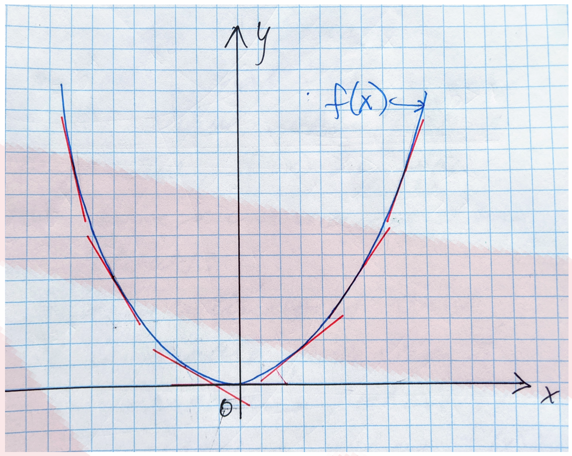
Increasing and Decreasing Functions

We say a function f is *decreasing* on an interval if for $x_1 < x_2, f(x_1) > f(x_2)$.
We say a function f is *increasing* on an interval if for any $x_1 < x_2, f(x_1) < f(x_2)$

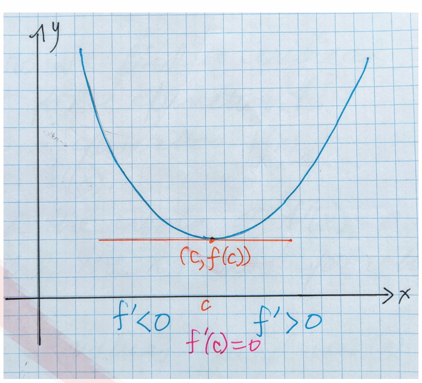


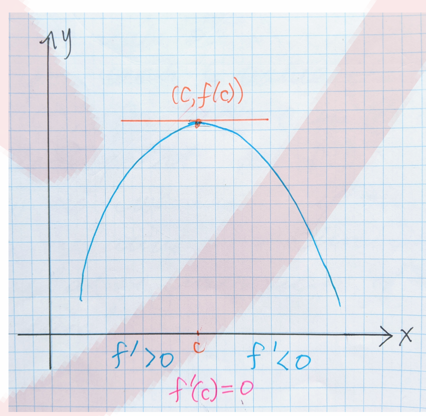
If we consider the graph above, when $x < 0$ the function is decreasing. Notice that the tangents to the curvce when it is decreasing are all negative or $f'(x) < 0$ for $x < 0$. When $x > 0$, the function is increasing. Notice that the tangents to the curve when $x > 0$ are all positive, $f'(x) > 0$, for $x > 0$. We can summaraize this as follows:

For a continuous and differentiable function f,
f is increasing when $f'(x) > 0$ and f is decreasing when $f'(x) < 0$.

Maximum or Minimum

If we look at the graph below, we see that one graph has a maximum value and the other has a minimum value.





Notice in left figure we have a minimum value when $x = c$ or when $f'(c) = 0$ The point where this minimum occurs is $(c, f'(c))$. In the right hand figure we have a similar situation except now we have a maximum value when $x = c$ or at point $(c, f(c))$. Again $f'(c) = 0$. This point $(c, f(c))$ is called a *critical point* of the function f.

Critical points

How do we find a critical point for a function f? For a function $f(x)$

1. Find $f'(x)$.
2. Find the roots of $f'(x) = 0$. So, find the x values for which $f'(x) = 0$.
3. The x-values found in 2. can be labeled c. Evalutate $f(c)$ for each c value in 2.
4. $(c, f(c))$, for each c in 2. is a *critical point* for f.

Relative maximum or minimum?

How do we know if $f(c)$ is a relative maximum or relative minimum of the function?

1. For a function f where $(c, f(c))$ is a critical point, if $f(c) < f(x)$ for x near c then $f(c)$ is a local minimum values. If $f(c) > f(x)$ for x near c then $f(c)$ is a local maximum value.
2. Another way to determine if a critical point c give a maximum or minimum value is to consider the second derivative. If $f''(c) > 0$ then c gives a minimum value, that is $f(c)$ is a minimum; if $f''(c) < 0$ then c gives a maximum value, that is $f(c)$ is a maximum.

Exercises

1. Find the critical points for the following.

   a) $y = x^3 - 6x^2 - 15x + 10$

   b) $y = \frac{25}{x^2+48}$

   c) $y = x + x^{-1}$

   d) $y = (x-3)^3 + 8$

2. For #1 determine if the critical point is a local maxium, local minimum or neither using the second derivative.

3. Determine the point of inflection for each function in #1.