Implicit Differentiation 4

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What is implicit differentiation?

It's common to be face with functions of the form $y = f(x)$ and then differentiate y w.r.t. x. This is *explicit* differentiation. However, functions or relations of the form $x^2 + y^2 = 16$ can also be differentiated without solving for y. This is referred to as *implicit* differentiation. Let's consider an example.

Example

If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$.

Solution:

Step 1. Differentiate both sides of $x^2 + y^2 = 25$ w.r.t. x.

$$\begin{aligned} \frac{d}{dx}(x^2 + y^2) &= \frac{d}{dx}(25) \\ 2x + 2y\frac{dy}{dx} &= 0 \end{aligned}$$

Step 2. Solve for $\frac{dy}{dx}$.

$$\begin{aligned} \frac{2x + 2y\frac{dy}{dx}}{2} &= \frac{0}{2} \\ x + y\frac{dy}{dx} &= 0 \\ y\frac{dy}{dx} &= -x \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

Example

Let's consider another example. Find $\frac{dy}{dx}$ given $2xy - y^3 = 4$.

Solution: Start by differentiating both sides.

$$\begin{aligned}
\frac{d}{dx}(2xy - y^3) &= \frac{d}{dx}(4) \\
\frac{d}{dx}(2xy) - \frac{d}{dx}(y^3) &= 0, \quad \text{use the Chain rule} \\
2y + (2x - 3y^2)\frac{dy}{dx} &= 0, \quad \text{Solve for } \frac{dy}{dx} \\
(2x - 3y^2)\frac{dy}{dx} &= -2y \\
\frac{dy}{dx} &= \frac{-2y}{2x - 3y^2}
\end{aligned}$$

Procedure for implicit differentiation

Let's summaraize the procedure for implicit differentiation.

1. You have an equation defined implicitly.
2. Differentiate both sides w.r.t. x. Use the chain rule when needed.
3. Solve for $\frac{dy}{dx}$.

Exercises

Use implicit differentiation to find $\frac{dy}{dx}$.

a) $x^2 + y^2 = 36$

b) $15y^2 = 2x^3$

c) $3xy^2 + y^3 = 8$

d) $9x^2 - 16y^2 = -144$

e) $3x^2 + 4xy^3 = 9$

f) $x^3y^3 = 144$

g) $x = y + y^5$

h) $xy^3 - x^3y = 2$

i) $\sqrt{x} + \sqrt{y} = 5$