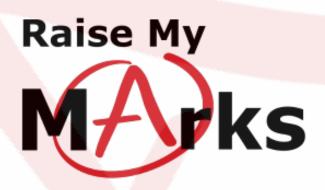
Implicit Differentiation 4



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What is implicit differentiation?

It's common to be face with functions of the form y = f(x) and then differentiate y w.r.t. x. This is *explicit* differentiation. However, functions or relations of the form $x^2 + y^2 = 16$ can also be differentiated without solving for y. This is referred to as *implicit* differentiation. Let's consider an example.

Example

If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$.

Solution:

Step 1. Differentiate both sides of $x^2 + y^2 = 25$ w.r.t. x.

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$
$$2x + 2y\frac{dy}{dx} = 0$$

Step 2. Solve for $\frac{dy}{dx}$.

$$\frac{2x + 2y\frac{dy}{dx}}{2} = \frac{0}{2}$$
$$x + y\frac{dy}{dx} = 0$$
$$y\frac{dy}{dx} = -x$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

Example

Let's consider another example. Find $\frac{dy}{dx}$ given $2xy - y^3 = 4$.



Solution: Start by differentiating both sides.

$$\frac{d}{dx}(2xy - y^3) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}(2xy) - \frac{d}{dx}(y^3) = 0, \text{ use the Chain rule}$$

$$2y + (2x - 3y^2)\frac{dy}{dx} = 0, \text{ Solve for } \frac{dy}{dx}$$

$$(2x - 3y^2)\frac{dy}{dx} = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{2x - 3y^2}$$

Procedure for implicit differentiation

Let's summaraize the procedure for implicit differentiation.

- 1. You have an equation defined implicitly.
- 2. Differentiate both sides w.r.t. x. Use the chain rule when needed.
- 3. Solve for $\frac{dy}{dx}$.



Exercises

Use implicit differentiation to find $\frac{dy}{dx}$.

a)
$$x^2 + y^2 = 36$$
 f) $x^3y^3 = 144$

b) $15y^2 = 2x^3$

g) $x = y + y^5$

c)
$$3xy^2 + y^3 = 8$$

h) $xy^3 - x^3y = 2$

d)
$$9x^2 - 16y^2 = -144$$

i) $\sqrt{x} + \sqrt{y} = 5$

e) $3x^2 + 4xy^3 = 9$