Graph Sketching 2

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Graph Sketching

Using everything you have learned about the derivative of a function, we're going to put it all together to sketch the graph of the curve represented by a function.

Algorithm for graph sketching

1. Find discontinuities of $f(x)$. e.g. when $f(x) = \pm\infty$.
2. Find the asymptotes, vertical, horizontal and oblique.
3. Determin the intercepts, y and x intercepts.
4. Determine the critical pionts from $f'(x) = 0$.
5. Determinen concavity using $f''(x)$.
6. Determine inflection points from $f''(x) = 0$.
7. Sketch graph using above information.

Example

Sketch $y = x^3 - 3x^2 - 9x + 10$

Solution:

$$\begin{aligned} y' &= 3x^2 - 6x - 9 \\ y'' &= 6x - 6 \\ y = 0 &= 3x^2 - 6x - 9 \\ 0 &= x^2 - 2x - 3 \\ 0 &= (x-3)(x+1) \end{aligned}$$

Therefore, $c = 3 - 1$.

$$\begin{aligned} y''(3) &= 18 - 6 = 12 > 0 \\ y''(-1) &= -6 - 6 = -12 < 0 \end{aligned}$$

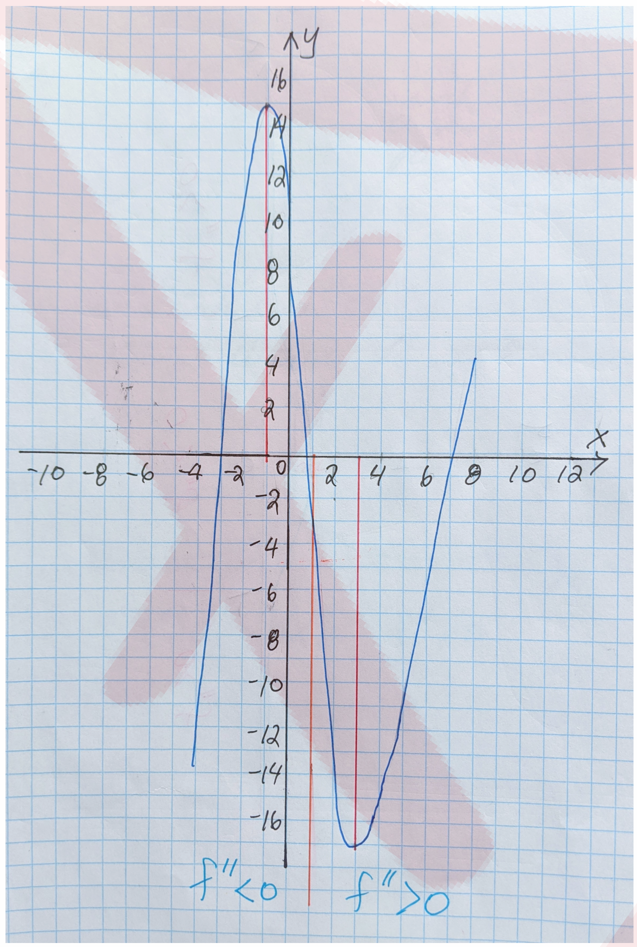
Therefore the function is concave up at $c = 3$ and concave down at $c = -1$. Where does the function change concavity, or where is the inflection point?

$$y'' = 6x - 6 = 0 \Rightarrow x = 1$$

Therefore when $x = 1 = c$ we have an inflection point. Let's now find the points at which the function has a minimum, maximum and inflection point.

$$\begin{aligned} y(3) &= 27 - 27 - 27 + 10 = -17 \\ y(-1) &= -1 - 3 + 9 + 10 = 15 \\ y(1) &= 1 - 3 - 9 + 10 = -1 \end{aligned}$$

Therefore, $(3, -17), (-1, 15)$ and $(1, -1)$ are the minimum, maximum and inflection points, respectively, of the function f.



Exercises

Sketch each of the following,

g) $c = te^{-t} + 5$

h) $g(x) = \frac{8e^x}{e^{2x}+4}$

i) $f(x) = \frac{k-x}{k^2+x^2}$

j) $g(x) = x^{1/3}(x+3)^{2/3}$

k) $f(x) = \frac{x}{\sqrt{x^2+1}}$