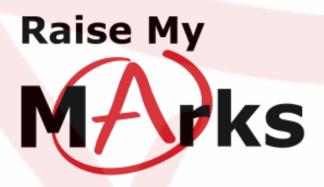
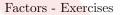
Factors



RaiseMyMarks.com

2020





What is a factor of a function?

Specifically, a polynomial function? A linear factor x - a of a function f(x) is such that f(a) = 0. For example, show that x - 2 is a factor of $x^3 - 3x^2 + 5x - 6$. Let a = 2. Then we have,

$$f(x) = x^{3} - 3x^{2} + 5x - 6$$

$$f(2) = 8 - 12 + 10 - 6$$

$$= -4 + 4$$

$$= 0$$

or x - 2 is a factor of f(x) when the remainder of $f(x) \div (x - 2)$ is 0. Let's check this is true.

$$\begin{array}{r} x^{2} - x + 3 \\ x - 2 \overline{\smash{\big)}} x^{3} - 3x^{2} + 5x - 6 \\ \underline{-(x^{3} - 2x^{2})} \\ \hline -x^{2} + 5x \\ -\underline{(-x^{2} + 2x)} \\ 3x - 6 \\ \underline{-(3x - 6)} \\ 0 \end{array}$$

So the remainder r = 0. Therefore, x - 2 is a facor of $x^3 - 3x^2$. This is summarized nicely in the **Factor Theorem**,

Factor Theorem

$$x - p$$
 is a factor of $f(x)$ if and only if $f(p) = 0$.

Some special polynomials and factors:

The sum and difference of cubes:

$$x^{3} - a^{3} = (x - a)(x^{2} + xa + a^{2})$$

 $x^{3} + a^{3} = (x + a)(x^{2} - xa + a^{2})$

1



Difference of squares

$$x^{2} - a^{2} = (x - a)(x + a)$$

Quadratic factors

$$ax^{2} + bx + c = \left(x - \frac{-b + \sqrt{b^{2} - 4ac}}{2a}\right) \left(x - \frac{-b - \sqrt{b^{2} - 4ac}}{2a}\right)$$

where the quadratic formula has been used to find the roots and hence the factors of the quadratic. In general a polynomial of degree n has at most n real roots. It

is possible for a polynomial of degree n to have less than n real roots but every polynomial of degree nn has exactly n roots, real or complex.

Some properties of Quadratics

There are some interesting properties of quadratics. Let's consider the quadratic

$$q(x) = ax^2 + bx + c,$$

where a, b, c are real numbers.

- 1. The sum of the roots of $q(x) = 0 = ax^2 + bx + c$ is $-\frac{b}{a}$.
- 2. The products of the roots of $q(x) = 0 = ax^2 + bx + c$ is $\frac{c}{a}$.
- 3. Any quadratic equation can be written as,

$$0 = x^{2} - (\text{ sum of roots })x + (\text{ product of roots })$$

$$0 = ax^{2} + bx + c$$

$$0 = a\left(x^{2} + \frac{b}{a}x + \frac{c}{a}\right)$$

$$0 = a\left(x^{2} - \left(-\frac{b}{a}\right)x + \frac{c}{a}\right)$$



Note: Factors of functions are not necessarily just linear. The following function has quadratic factors,

$$f(x) = (x^2 + 1)^2$$

You may think of a factor of a function f(x) as any functions $g_1(x), \ldots, g_n(x)$ such that $f(x) = g_1(x) \cdots g_n(x)$, where at least two of the functions $g_i(x)$ and $g_j(x)$ for $i \neq j$ are NOT equal to 1. Mos the functions we will consider will have factors that are linear or polynomial.

Exercises

- 1. For the given linear function and function f(x), is the linear function and factor of f(x)? Explain why or why not.
 - a) x 1, $f(x) = x^2 7x + 6$

b) x - 3, $f(x) = x^3 + 6x^2 - 2x + 3$

c) x + 1, $f(x) = x^3 - 5x^2 - 4x + 3$

d) 2x - 1, $f(x) = 4x^3 - 6x^2 + 8x - 3$



e)
$$x - 2$$
, $f(x) = x^3 - 3x^2 - 4x + 12$

- 2. If (x 1) is a factor of $x^3 2kx^2 + 3x + 1$, what is k?
- 3. Show x y is a factor of $x^4 y^4$.