Exponential Growth and Decay 2

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Exponential growth and decay

This is a common application of the exponential function. Exponential growth/decay occurs when quantitites increase or decrease at a rate proportional to the quantit present. Some examples of where growth or decay occurs is in savings accounts, size of populations, decay of radioactive chemicals. Let's look at an example.

Example

The population of a city is 810 000. If it is increasing at 4% per year, estimate the population in four years.

Solution:

$$y = C(1 + 0.04)^t$$

$C = 810000 =$ initial population; $y =$ population after t years. Therefore we have,

$$\begin{aligned} y &= 810000(1.04)^t \\ y(4) &= 810000(1.04)^4 \\ &= 947585.4 \end{aligned}$$

So the population after 4 years is approximately 947586.

Example

A used car dealer sells a five year old car for \$4200. What was the original value of the car if the depreciation is 15% a year?

Solution:

$$y = Cb^t$$

where $b = 1 - 0.15 = 0.85$

$$\begin{aligned} y(5) = 4200 &= C(0.85)^5 \\ \frac{4200}{(085)^5} &= C \\ \$9465.74 &= C \end{aligned}$$

Therefore, the original price of the car is \$9465.74.

Example

A bacteria population doubles in 5 days. When will it be 16 times as large?

Solution:

$$y = C2^{t/5}$$

where C = initial population and y = population after t days.

$$\begin{aligned} y &= C2^{t/5} \\ \frac{16C}{C} &= \frac{C}{C}2^{t/5} \\ 16 &= 2^{t/5} \\ 2^4 &= 2^{t/5} \\ 4 &= t/5 \\ 20 &= t \end{aligned}$$

Therefore, after 20 days the population will be 16 times as great as the initial population.

Example

A research assistant made 160mg of radioactive sodium Na^{24} and found that there was only 20mg left after 45 hours. What is the half life of Na^{24}?

Solution: We have $C = 160$, '$y(45) = 20$ and we want to find $b =?$.

$$\begin{aligned} y(t) &= Cb^t \\ y(t) &= C\left(\frac{1}{2}\right)^t \\ 20 &= 160\left(\frac{1}{2}\right)^{45k} \\ \frac{20}{160} &= \frac{160}{160}\left(\frac{1}{2}\right)^{45k} \\ \frac{1}{8} &= \left(\frac{1}{2}\right)^{45k} \\ \frac{1}{2^3} &= \left(\frac{1}{2}\right)^{45k} \\ \frac{3}{45} &= \frac{45k}{45} \\ \frac{1}{15} &= k \end{aligned}$$

Therefore,

$$\begin{aligned} y(t) &= 160\left(\frac{1}{2}\right)^{t/15} \\ \frac{80}{160} &= \frac{160}{160}\left(\frac{1}{2}\right)^{t/15} \\ \frac{1}{2} &= \left(\frac{1}{2}\right)^{t/15} \\ 1 &= \frac{t}{15} \\ 15 &= t \end{aligned}$$

Therefore, the half life is 15 hours.

Exercises

1. The first year of a charity walk even had 6000 participans. The attendance increases by 5% each year.
   a) Write a function that represents the situation described above. Is the situation exponential growth or decay?
   b) How many people will attend in the 5^{th} year?
2. The popluation of a small town was 3000 in 1980. The popluation inreases by 4% annually.
   a) Does the situation above represent exponential growth or decay? Write a function that represents the situation.
   b) What is the popluation in 2020?
3. Your starting salary at a new compan is $62500 and it increases by 3% each year.
   a) Does the situation above represent exponential growth or decay? Write a function that represents the situation.
   b) What will your salary be in 5 years? In 15 years?
4. The yearly revenue of a company is $25000. The revenue has been decreasing by 3% each month.
   a) What is the monthly revenue, assuming equal rvenue each month?
   b) Does the situation above represent exponential growth or decay? Write a function that represents the situation.
   c) Whas is the monthly revenue after 5 months of Covid-19 *shut down*?
   d) What is the monthly revenue after 5 months with no Covid-19 shut down?
5. A piece of land was purcahsed for $65000. The value of the land has been increasing by 1.5% annually.
   a) Does the situation above represent exponential growth or decay? Write a function that represents the situation.
   b) What is the value of the land after 50 years?