Derivative of Natural Logarithm

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Derivative of the natural logarithm function

Let's start with $y = \ln x$. We know,

$$y = \ln x \Leftrightarrow e^y = x$$

Differentiating both sides we have,

$$\begin{aligned} y'e^y &= 1 \\ y' &= \frac{1}{3} \\ &= \frac{1}{x} \end{aligned}$$

Therefore, we have,

$$\boxed{\text{If } f(x) = \ln x \text{ then } f'(x) = \frac{1}{x},\ x > 0.}$$

In general,

$$\boxed{f(x) = \ln(g(x)) \Rightarrow f'(x) = \frac{g'(x)}{g(x)}}$$

Let's consider a more general logarithmic function with base b and a function $g(x)$ in the exponent.

$$\begin{aligned} f(x) = b^{g(x)} &\quad\Rightarrow\quad f'(x) = g'(x)b^{g(x)}\ln b \\ f(x) = b^x &\quad\Rightarrow\quad f'(x) = b^x \ln b \\ y = \log_b x &\quad\Rightarrow\quad \frac{dy}{dx} = \frac{1}{x \ln b} \\ y = \ln x &\quad\Rightarrow\quad \frac{dy}{dx} = \frac{1}{x} \\ y = \log_b g(x) &\quad\Rightarrow\quad \frac{dy}{dx} = \frac{g'(x)}{g(x)\ln b} \end{aligned}$$

Therefore, we have,

$$\boxed{y = \log_b g(x) \Rightarrow \frac{dy}{dx} = \frac{g'(x)}{g(x)\ln b}}$$

Exercises

Differentiate the following,

a) $y = e^{x^2 \ln x}$

b) $y = \ln(x^2 - 2x + 3)$

c) $y = \ln(4x^3 + x)$

d) $f(x) = \ln(\sqrt{5x - 7})$

e) $f(x) = \frac{\ln x}{x}$

f) $f(x) = x \ln x - x$

g) $y = x \ln\left(\frac{1}{x}\right)$

h) $y = \ln(x^2 - 2x)$

i) $y = \frac{1}{\ln x}$