

Derivative of Exponential Function

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## Derivatives of the exponential function

### Derivative of $e^x$

Let's use the definition of the derivative to find the derivative of  $f(x) = b^x$ .

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{b^{x+h} - b^x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{b^x(b^h - 1)}{h} \\
 &= b^x \lim_{h \rightarrow 0} \frac{b^h - 1}{h} \\
 f'(x) &= b^x f'(0)
 \end{aligned}$$

Therefore,

$$f'(x) = b^x f'(0)$$

Let's consider the value of  $b = e$  where  $e = 2.718281\dots$

$$\text{For } f(x) = e^x, f'(x) = e^x.$$

In general,

$$f(x) = e^{g(x)} \Rightarrow f'(x) = g'(x)e^{g(x)}$$

## Exercises

Differentiate the following.

a)  $y = e^{3x}$

g)  $h(t) = e^{t^2} + 3e^{-t}$

b)  $s = e^{3t-5}$

h)  $g(t) = \frac{e^{2t}}{1+e^{2t}}$

c)  $y = e^{\sqrt{x}}$

i)  $f(x) = \frac{1}{3}(e^{3x} + e^{-3x})$

d)  $y = 2e^{x^2}$

j)  $f(x) = e^{-\frac{1}{x+1}}$

e)  $f(x) = \frac{e^{-x^3}}{x}$

k)  $y = \frac{5}{2}(e^{x/5} + e^{-x/5})$

f)  $f(x) = \sqrt{x}e^x$

l)  $y - e^{xy} = 0$

m)  $x^2 e^y = 1$