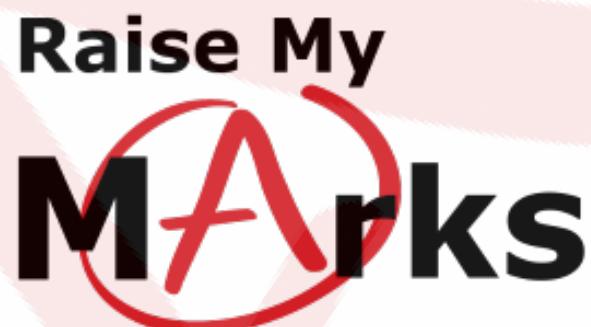


Derivative of Exponential Function



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Derivatives of the exponential function

Derivative of e^x

Let's use the definition of the derivative to find the derivative of $f(x) = b^x$.

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{b^{x+h} - b^x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{b^x(b^h - 1)}{h} \\
 &= b^x \lim_{h \rightarrow 0} \frac{b^h - 1}{h} \\
 f'(x) &= b^x f'(0)
 \end{aligned}$$

Therefore,

$$f'(x) = b^x f'(0)$$

Let's consider the value of $b = e$ where $e = 2.718281\dots$

$$\boxed{\text{For } f(x) = e^x, f'(x) = e^x.}$$

In general,

$$\boxed{f(x) = e^{g(x)} \Rightarrow f'(x) = g'(x)e^{g(x)}}$$

Exercises

Differentiate the following.

$$\text{a) } y = e^{3x}$$

$$\text{g) } h(t) = e^{t^2} + 3e^{-t}$$

$$\text{b) } s = e^{3t-5}$$

$$\text{h) } g(t) = \frac{e^{2t}}{1+e^{2t}}$$

$$\text{c) } y = e^{\sqrt{x}}$$

$$\text{i) } f(x) = \frac{1}{3}(e^{3x} + e^{-3x})$$

$$\text{d) } y = 2e^{x^2}$$

$$\text{j) } f(x) = e^{-\frac{1}{x+1}}$$

$$\text{e) } f(x) = \frac{e^{-x^3}}{x}$$

$$\text{k) } y = \frac{5}{2}(e^{x/5} + e^{-x/5})$$

$$\text{f) } f(x) = \sqrt{x}e^x$$

l) $y - e^{xy} = 0$

m) $x^2e^y = 1$