Concavity and point of inflection

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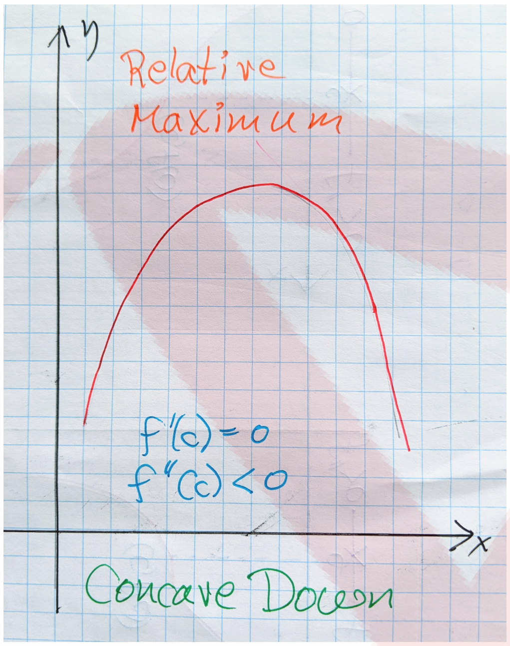
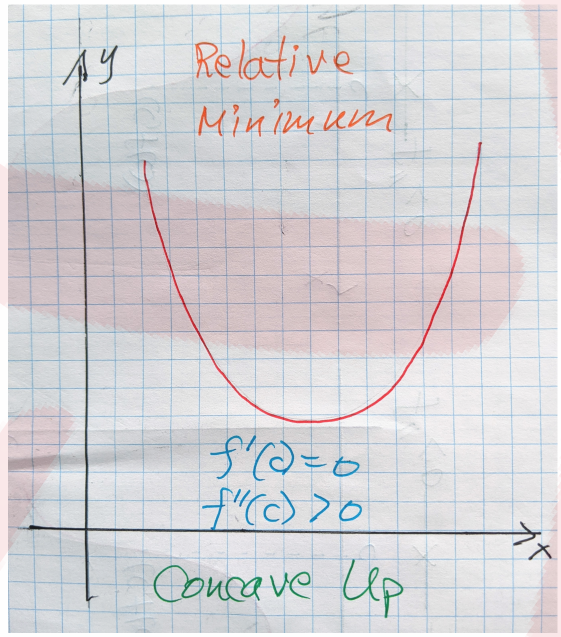
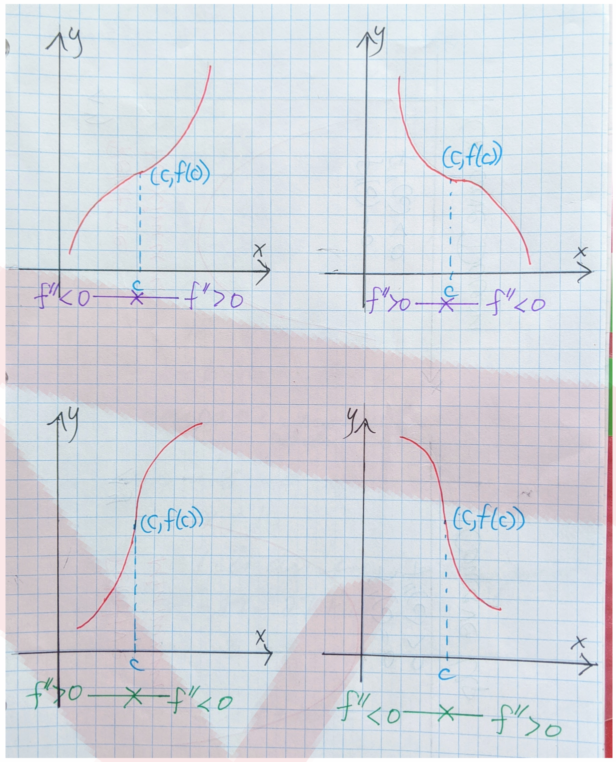
Concavity and points of inflection

At a critical point the function is a *relative maximum* or a *relative minimum.* The graph is said to be *concave down* at a *relative maximum* and *concave up* at a *relative minimum.*





A *point of inflection* is a critical point $(c, f(c))$ where $f''(c) = 0$. It is a point where th concavity of the function changes.



Let’s consider a few examples.

Example

Determine points of inflection for

$$f(x) = \frac{1}{x^2 + 3}$$

.

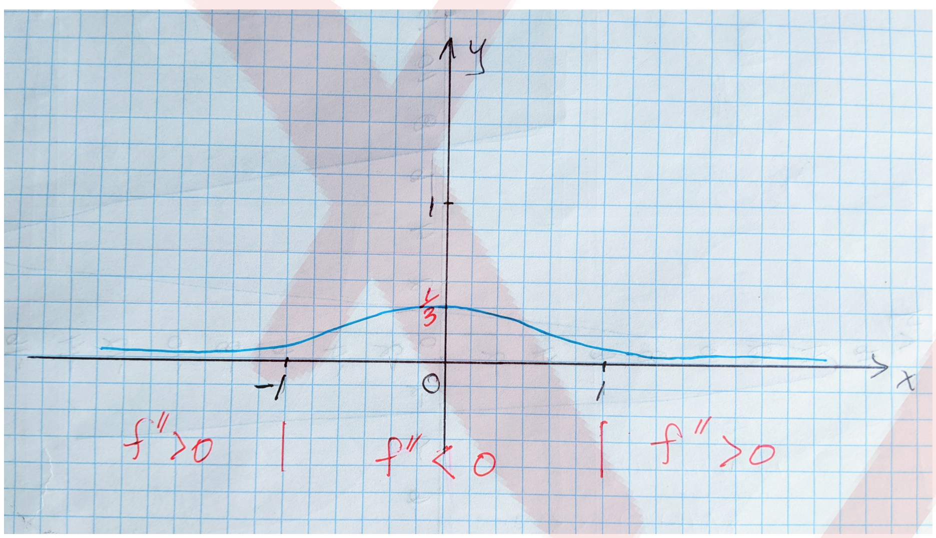
Solution:

$$\begin{aligned}
f'(x) &= -(x^2+3)^{-2}(2x) \\
&= \frac{-2x}{(x^2+3)^2} \\
f''(x) &= \frac{-2(x^2+3)^{+}(-2x)(2)(x^2+3)(2x)}{(x^2+3)^4} \\
&= \frac{-2(x^2+3)+8x^2}{(x^2+3)^3} \\
&= \frac{6x^2-6}{(x^2+3)^3}
\end{aligned}$$

Therefore, $x = +1$ or -1 are poins of inflection; $f'(x) = 0, x = 0$ is a critical point. We have,

$$f''(0) = -\frac{6}{3^3} = -\frac{6}{27} = -\frac{2}{9} < 0$$

Therefore, at $x = 0$ is a relative ,aximum or f is a concadve down at $x = 0$.



Example

Graph

$$f(x) = \frac{x-4}{x^2-x-2}$$

Soution:

$$f(x) = \frac{x-4}{x^2-x-2} = \frac{x-4}{(x-2)(x+1)}$$

Therefore, $x = 2, -1$ are discontinuities.

	-1	2	4	
$x-4$	-	-	-	+
$x-2$	-	-	+	+
$x+1$	-	+	+	+
$f(x)$	-	+	-	+

$$\begin{aligned}
\lim_{x\to -1^-} f(x) &= -\infty \\
\lim_{x\to -1^+} f(x) &= +\infty \\
\lim_{x\to 2^-} f(x) &= +\infty \\
\lim_{x\to 2^+} f(x) &= -\infty
\end{aligned}$$

Therefore, $x = 2, x = -1$ are vertical asymptotes.

$$\lim_{x\to\infty} \frac{x-4}{x^2-x-2} = \lim_{x\to\infty} \frac{x(1-4/x)}{x^2(1-1/x-2/x^2)} = 0$$

Therefore, $y = 0$ is a horiztonal asymptote.

Exercises

1. Find the second derivative for the following at the given point.

   a) $f(x) = 2x^3 - 10x + 3, \; x = 2$

   b) $g(x) = x^2 - \frac{1}{x} \; x = -1$

   c) $s = e^t \ln t, \; t = 1$

2. For each function in #1 determine the points of inflection.

3. For the functions in #1 determine whether the function lies above or below the function at the given point.