

Addition and Subtraction Formulas  
Trigonometry

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## Addition and Subtraction formulas

The triangles and related special angles,  $\pi/3$ ,  $\pi/6$ ,  $\pi/4$  have particular values for each of the three trigonometric ratios,  $\sin x$ ,  $\cos x$ ,  $\tan x$ . If a given angle can be written as the sum or difference of any of these special angles, using the addition and subtraction formulas below it is possible to find exact values to the trigonometric ratios for some angles. Let's first have a look at the the addition and subtraction formulas then we'll take a look at some examples.

### Addition and subtraction formuals for trigonometric functions

$$\sin(x + y) = \sin x \cos y + \cos x \sin y \quad (1)$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y \quad (2)$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \quad (3)$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y \quad (4)$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \quad (5)$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \quad (6)$$

### Double Angle Formulas

$$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x} \quad (7)$$

$$\cos(2x) = \cos^2 x - \sin^2 x \quad (8)$$

$$\sin(2x) = 2 \sin x \cos x \quad (9)$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad (10)$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \quad (11)$$

### Example

Find the exact value of  $\tan 75^\circ$ .

**Solution:**

$$\begin{aligned}
 \tan(75^\circ) &= \tan(45^\circ + 30^\circ) \\
 &= \frac{\tan(45^\circ) + \tan(30^\circ)}{1 - \tan(45^\circ)\tan(30^\circ)} \\
 &= \frac{1 + 1/\sqrt{3}}{1 - (1)(1/\sqrt{3})} \\
 &= \frac{(\sqrt{3} + 1)\sqrt{3}}{(\sqrt{3} - 1)\sqrt{3}} \\
 &= \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1}\right) \left(\frac{\sqrt{3} + 1}{\sqrt{3} + 1}\right) \\
 &= \frac{3 + 2\sqrt{3} + 1}{3 - 1} \\
 &= \frac{4 + 2\sqrt{3}}{2} \\
 &= 2 + \sqrt{3}
 \end{aligned}$$

Therefore,  $\tan(75^\circ) = 2 + \sqrt{3}$ .

**Example**

Show  $\tan(180 - x) = -\tan x$

**Solution:**

$$\begin{aligned}
 L.S. &= \tan(180 - x) \\
 &= \frac{\tan(180) - \tan x}{1 + \tan(180)\tan x} \\
 &= \frac{0 - \tan x}{1 + (0)\tan x} \\
 &= -\frac{\tan x}{1} \\
 &= -\tan x \\
 &= R.S.
 \end{aligned}$$

**Example**

Find  $\sin\left(\frac{7\pi}{12}\right)$

**Solution:** First let's try and rewrite the angle  $7\pi/12$  as the sum or difference of the special angles.

$$\frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi + 4\pi}{12} = \frac{7\pi}{12}$$

or in degrees,

$$\frac{7\pi}{12} = \frac{7(180)}{12} = 7 \times 15 = 75^\circ = 45^\circ + 30^\circ$$

Now, let's apply one of the addition formulas and see what value we get applying the special triangle/angle values.

$$\begin{aligned} \sin\left(\frac{7\pi}{12}\right) &= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \\ &= \sin\frac{\pi}{4} \cos\frac{\pi}{3} + \cos\frac{\pi}{4} \sin\frac{\pi}{3} \\ &= \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \left(\frac{\sqrt{3}}{2}\right) \\ &= \left(\frac{1 + \sqrt{3}}{2\sqrt{2}}\right) \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{(1 + \sqrt{3})\sqrt{2}}{2(\sqrt{2})} \\ &= \frac{(1 + \sqrt{3})\sqrt{2}}{4} \end{aligned}$$

### Example

Show  $\sin 2\theta = 2 \sin \theta \cos \theta$

**Solution:**

$$\begin{aligned} L.S. &= \sin 2\theta \\ &= \sin(\theta + \theta) \\ &= \sin \theta \cos \theta + \cos \theta \sin \theta \\ &= 2 \sin \theta \cos \theta \\ &= R.S. \end{aligned}$$

### Example

Given  $\tan A = -3$  find  $\tan(2A)$ .

Solution:

$$\begin{aligned}\tan(2A) &= \frac{2 \tan A}{1 - \tan^2 A} \\ &= \frac{2(-3)}{1 - (-3)^2} \\ &= \frac{-6}{1 - 9} \\ &= \frac{-6}{-8} \\ &= \frac{3}{4}\end{aligned}$$

### Exercises

1. Verify  $\tan(180 + x) = \tan x$ .

2. Show  $\tan(360 - x) = -\tan x$ .

3. Find the exact value of,

i)  $\tan(330)$

iii)  $\sin(330)$

v)  $\cos(15^\circ)$

ii)  $\cos(330)$

iv)  $\tan(15^\circ)$

vi)  $\sin(15^\circ)$

vii)  $\cos(75^\circ)$

xiv)  $\tan(120)$

xxi)  $\cos(225)$

viii)  $\sin(75^\circ)$

xv)  $\cos(120)$

xxii)  $\sin(225)$

ix)  $\cos(90 + x)$

xvi)  $\sin(120)$

xxiii)  $\tan(135)$

x)  $\sin(90 - x)$

xvii)  $\tan(150)$

xxiv)  $\cos(135)$

xi)  $\tan(105)$

xviii)  $\cos(150)$

xxv)  $\sin(135)$

xii)  $\cos(105)$

xix)  $\sin(150)$

xiii)  $\sin(105)$

xx)  $\tan(225)$

xxvi)  $\tan(270)$

xxix)  $\cos(90 + x)$

xxxii)  $\sin(270 + x)$

xxvii)  $\cos(270)$

xxx)  $\cos(270 - x)$

xxxiii)  $\sin(270 - x)$

xxviii)  $\sin(270)$

xxxi)  $\cos(270 + x)$

4. Find the exact value for the following,

a)  $\sin 20 \cos 40 + \cos 20 \sin 40$

e)  $\cos 55 \cos 35 - \sin 55 \sin 35$

b)  $\cos 100 \sin 40 + \sin 100 \cos 40$

f)  $\cos 130 \cos 85 + \sin 130 \sin 85$

c)  $\sin 80 \cos 35 - \cos 80 \sin 35$

g)

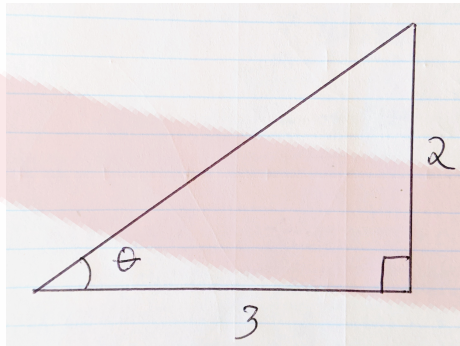
$$\frac{\tan 25 + \tan 35}{1 + \tan 25 \tan 35}$$

d)  $\sin 10 \cos 50 + \cos 10 \sin 50$

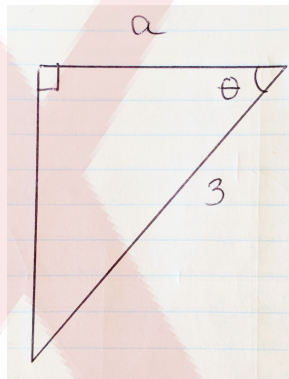
h)

$$\frac{\tan 170 - \tan 125}{1 + \tan 170 \tan 125}$$

5. Using the diagram below, find  $\sin \theta$ .



6. Find  $\cos \theta$  in term of  $a$ .



7. Given  $\cos \theta = \frac{x}{4}$  find an expression for  $\cos 2\theta$ .

8. Solve  $\sin 2x - \cos x$ ,  $0 \leq x \leq 2\pi$ .



9. Find the following values using the double angle formula

a)  $\sin(120)$

b)  $\cos(90)$

10. Verify  $\cos(2\theta) = 1 - 2\sin^2\theta$ .

11. Show

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

12. Suppose  $A$  is in quadrant 1 and  $\cos A = \frac{3}{5}$ . Find  $\sin(3A)$ .

13. Suppose  $\sin \phi = \frac{1}{\sqrt{2}}$ . Find  $\cos 2\phi$ .