Rational Functions & Oblique Asymptotes

RaiseMyMarks.com

2020

Rational Functions

Not all asymptotes are vertical or horizontal lines. It is possible for an asymtote to be a line with a negative or positive slope. These asymptotes have the equation of a line of the form $y = mx + b$ where $m \neq 0$. An asymptote of this form is called an *oblique asymptote*. So, how do we find an oblique asymptote? First,

How do we know a rational functions has an oblique asymtote?

When we are faced with a rational function of the form,

$$f(x) = \frac{g(x)}{h(x)}$$

where $p(x)$ and $q(x)$ are polynomials, if the degree of $g(x)$ is one more than the degree of $h(x)$, then $f(x)$ is a candidate for an oblique asymptote. So,

$$\text{degree } (\ g(x)\ ) = \text{degree } (h(x)) + 1$$

then $f(x)$ may have an oblique asymptote.

How do we find an oblique asymptote?

To find an oblique asymptote we need to do long division of polynomials, $h(x)$ into $g(x)$. If we get a remainder function $r(x)$ then the quotient function $q(x)$ is our oblique asymptote. That is,

$$f(x) = \frac{g(x)}{h(x)} = q(x) + \frac{r(x)}{h(x)}$$

then $y = q(x)$ is the oblique asymptote for the function $f(x)$. Let's try an example.

Example

Find the oblique asymptote for the following rational functions,

$$\frac{x^2 - 2x - 5}{x - 1}$$

©Raise My Marks 2020

Solution:

$$
\begin{array}{r|l}
 & x - 1 \\
\hline
x - 1 & x^2 - 2x - 5 \\
 & -(x^2 - x) \\
\hline
 & -x - 5 \\
 & -(-x + 1) \\
\hline
 & -6
\end{array}
$$

Therefore, $y = x - 1$ is the oblique asymptote.

©Raise My Marks 2020

Exercises

Find the oblique asymptote for the following rational functions,

a) $y = \frac{-2x^2+3x+1}{x+2}$

b) $y = \frac{x^2+x-6}{2-x}$

c) $y = \frac{x^2-4x+4}{x-1}$

d) $y = \frac{x^2+2x-15}{3-x}$

e) $f(x) = \frac{2x^3+x^2+11x+5}{x^2+5}$

f) $f(x) = \frac{2x^2-1+3x^3}{3-2x^2}$

g) $f(x) = \frac{2x^2+x+1}{8x}$

h) $f(x) = \frac{x^2+6x}{x+2}$

i) $f(x) = \frac{-3x^3+x+1}{x^2+x+2}$

j) $f(x) = \frac{x^2+1}{x}$

©Raise My Marks 2020