L'Hôpital's Rule

Indeterminate forms of Limits 3

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Indeterminate forms of Limits

It is possible when evaluating a limit we run into some problems. For example, when we go through the steps of evaluating the limit and we end up in one of the following situations,

$$\frac{\infty}{\infty},\ \frac{0}{0},\ (0)(\pm\infty),\ 1^{\infty},\ 0^{0},\ \infty^{0},\ \infty-\infty \tag{1}$$

Any of these forms in (1) is referred to as an *indeterminate form* of a limit. When faced with any of these indeterminate forms the following "rule" is performed to hopefully eliminate the indeterminate form. This rule is called **l'Hôpital's Rule** given below,

l'Hôpital's Rule

Suppose,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}\text{ or }\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

where a is any real number ∞ or $+\infty$. Then,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Example

Find the following limit

$$\lim_{x\to 0}\frac{\sin x}{x}$$

Solution:

$$\begin{aligned}&\lim_{x\to 0}\frac{\sin x}{x}\\ =\ &\frac{0}{0},\ \text{indeterminatne form so apply l'Hôpital's rule}\\ =\ &\lim_{x\to 0}\frac{\cos x}{1}\\ =\ &1\end{aligned}$$

Example

Evaluate the following limit,

$$\lim_{x\to 0^{+}} x\ln x$$

Solution:

$$\begin{aligned} & \lim_{x\to 0^+} x\ln x \\ = & \ 0\cdot\infty \end{aligned}$$

While $0\cdot\infty$ is an indeterminate form it is not in one of the forms that is found in l'Hôpital's rule. So we need to try and rearrange the function so that we obtain either $\frac{0}{0}$ or $\frac{\infty}{\infty}$ as the indeterminate form.

$$\begin{aligned} & \lim_{x\to 0^+} x\ln x \\ = & \lim_{x\to 0^+} \frac{\ln x}{1/x} \text{ now we will evaluate the limit of this form of the function ad see what we get.} \\ = & \lim_{x\to 0^+} \frac{1/x}{-1/x^2} \\ = & \frac{\infty}{\infty} \text{ Now we can apply l'Hôpital's Rule} \\ = & \lim_{x\to 0^+} \frac{1}{x}\left(-\frac{x^2}{1}\right) \\ = & \lim_{x\to 0^+} (-x) \\ = & \ 0 \end{aligned}$$

Note:

$$f(x)g(x) = \frac{g(x)}{1/f(x)} = \frac{f(x)}{1/g(x)}$$

Example

Evaluate the following limit,

$$\lim_{x\to a} \frac{x-a}{x^2-a^2}$$

Solution:

$$\begin{aligned} = & \lim_{x\to a} \frac{x-a}{x^2-a^2} \\ = & \lim_{x\to a} \frac{x-a}{(x-a)(x+a)} \\ = & \lim_{x\to a} \frac{1}{x+a} \\ = & \frac{1}{2a} \end{aligned}$$

Exercises

Find the following limits,

1. $$\lim_{x \to \pi/2} \frac{\cos x}{\pi/2 - x}$$

2. $$\lim_{x \to 0} frace^x - x - 1x^2$$

3. $$\lim_{x \to 0} \frac{\tan x}{\sqrt{x}}$$

4. $$\lim_{x \to 0} (x+1)^{1/x}$$

5. $$\lim_{x \to 0} \frac{\sin x - x}{x^2}$$

6. $$\lim_{x \to 0^+} x \ln(x^4)$$

7. $$\lim_{x \to 1^+} \left(\frac{1}{\ln x} - \frac{1}{1-x} \right)$$

8. $$\lim_{x \to \infty} \left(\frac{x+7}{x+3} \right)^x$$

9.
$$\lim_{x\to 0} \frac{3^x - 2^x}{x}$$