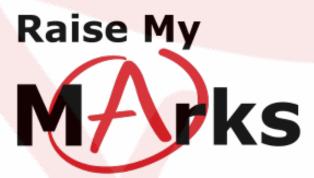
L'Hôpital's Rule Indeterminate forms of Limits 2



 ${\bf Raise My Marks. com}$

2020



Indeterminate forms of Limits

It is possible when evaluating a limit we run into some problems. For example, when we go through the steps of evaluating the limit and we end up in one of the following situations.

$$\frac{\infty}{\infty}, \ \frac{0}{0}, \ (0)(\pm \infty), \ 1^{\infty}, \ 0^0, \ \infty^0, \ \infty - \infty$$
 (1)

Any of these forms in (1) is referred to as an *indeterminate form* of a limit. When faced with any of these indeterminate forms the following "rule" is performed to hopefully eliminate the indeterminate form. This rule is called **l'Hôpital's Rule** given below,

l'Hôpital's Rule

Suppose,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm \infty}{\pm \infty}$$

where a is any real number ∞ or $+\infty$. Then,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Example

Find the following limit

$$\lim_{x \to 0} \frac{\sin x}{x}$$

Solution:

$$\lim_{x\to 0} \frac{\sin x}{x}$$

$$= \frac{0}{0}, \text{ indeterminate form so apply l'Hôpital's rule}$$

$$= \lim_{x\to 0} \frac{\cos x}{1}$$

$$= 1$$

Example

Evaluate the following limit,

$$\lim_{x \to 0^+} x \ln x$$



Solution:

$$\lim_{x \to 0^+} x \ln x$$
$$= 0 \cdot \infty$$

While $0 \cdot \infty$ is an indeterminate form it is not in one of the forms that is found in l'Hôpital's rule. So we need to try and rearrange the function so that we obtain either $\frac{0}{0}$ or $\frac{\infty}{\infty}$ as the indeterminate form.

$$\lim_{x \to 0^+} x \ln x$$

 $=\lim_{x\to 0^+}\frac{\ln x}{1/x}$ now we will evaluate the limit of this form of the function ad see what we get.

$$= \lim_{x \to 0^+} \frac{1/x}{-1/x^2}$$

 $=\frac{\infty}{\infty}$ Now we can apply l'Hôpital's Rule

$$= \lim_{x \to 0^+} \frac{1}{x} \left(-\frac{x^2}{1} \right)$$
$$= \lim_{x \to 0^+} (-x)$$

Note:

$$f(x)g(x) = \frac{g(x)}{1/f(x)} = \frac{f(x)}{1/g(x)}$$

Example

Evaluate the following limit,

$$\lim_{x \to a} \frac{x - a}{x^2 - a^2}$$

Solution:

$$= \lim_{x \to a} \frac{x - a}{x^2 - a^2}$$

$$= \lim_{x \to a} \frac{x - a}{(x - a)(x + a)}$$

$$= \lim_{x \to a} \frac{1}{x + a}$$

$$= \frac{1}{2a}$$



Exercises

Find the following limits,

1.

$$\lim_{x \to 0} \frac{\sin x}{x}$$

5.

$$\lim_{x \to \infty} x^{1/x^2}$$

2.

$$\lim_{x \to \infty} \frac{\ln x}{x^k}$$

6.

$$\lim_{x \to 0} x^{2/x}$$

3.

$$\lim_{x \to a} \frac{x - a}{x^2 - a^2}$$

7.

$$\lim_{x \to 0} \frac{x^2}{1/x}$$

1

$$\lim_{x \to 0^+} x^2 \ln x$$

8.

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$