L'Hôpital's Rule Indeterminate forms of Limits



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2020



## Indeterminate forms of Limits

It is possible when evaluating a limit we run into some problems. For example, when we go through the steps of evaluating the limit and we end up in one of the following situations,

$$\frac{\infty}{\infty}, \ \frac{0}{0}, \ (0)(\pm\infty), \ 1^{\infty}, \ 0^0, \ \infty^0, \ \infty - \infty$$
(1)

Any of these forms in (1) is referred to as an *indeterminate form* of a limit. When faced with any of these indeterminate forms the following "rule" is performed to hopefully eliminate the indeterminate form. This rule is called **l'Hôpital's Rule** given below,

## l'Hôpital's Rule

Suppose,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm \infty}{\pm \infty}$$

where a is any real number  $\infty$  or  $+\infty$ . Then,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

#### Example

Find the following limit

$$\lim_{x \to 0} \frac{\sin x}{x}$$

Solution:

$$\lim_{x \to 0} \frac{\sin x}{x}$$

$$= \frac{0}{0}, \text{ indetermination form so apply l'Hôpital's rule}$$

$$= \lim_{x \to 0} \frac{\cos x}{1}$$

$$= 1$$

#### Example

Evaluate the following limit,

 $\lim_{x \to 0^+} x \ln x$ 



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Solution:

$$\lim_{x \to 0^+} x \ln x$$
$$= 0 \cdot \infty$$

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While  $0 \cdot \infty$  is an indeterminate form it is not in one of the forms that is found in l'Hôpital's rule. So we need to try and rearrange the function so that we obtain either  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$  as the indeterminate form.

$$\lim_{x \to 1} x \ln x$$

- $x \rightarrow 0^+$
- =  $\lim_{x\to 0^+} \frac{\ln x}{1/x}$  now we will evaluate the limit of this form of the function ad see what we get.

$$= \lim_{x \to 0^+} \frac{1/x}{-1/x^2}$$

 $= \frac{\infty}{\infty}$  Now we can apply l'Hôpital's Rule

$$= \lim_{x \to 0^+} \frac{1}{x} \left( -\frac{x^2}{1} \right)$$
$$= \lim_{x \to 0^+} (-x)$$
$$= 0$$

Note:

$$f(x)g(x) = \frac{g(x)}{1/f(x)} = \frac{f(x)}{1/g(x)}$$

## Example

Evaluate the following limit,

$$\lim_{x \to a} \frac{x-a}{x^2 - a^2}$$

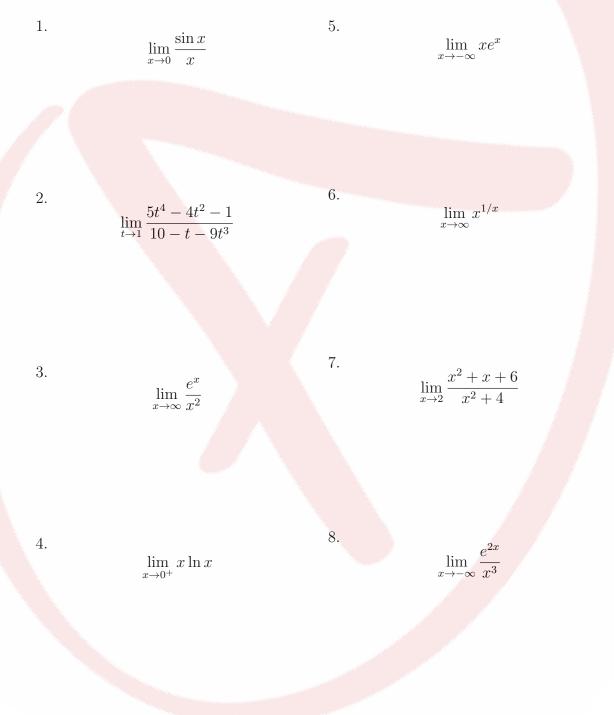
Solution:

$$= \lim_{x \to a} \frac{x-a}{x^2 - a^2}$$
$$= \lim_{x \to a} \frac{x-a}{(x-a)(x+a)}$$
$$= \lim_{x \to a} \frac{1}{x+a}$$
$$= \frac{1}{2a}$$

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# Exercises

Find the following limits,



l'Hôpital's Rule: Indeterminate forms of Limits - Exercises 9. 10.  $\lim_{x \to \infty} \frac{x + \cos x}{x}$  $\lim_{x\to\infty}\frac{e^x}{x^k}$ 4