Gemetric Series

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Geometric Series

A geometric series is the sum of the terms in a geometric sequence. Let's consider the geometric sequences and then sums.

$$\sum_1 = 3 + 6 + 12 + 24 + 48 + 96 + \cdots$$

$$\sum_2 = 2 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \cdots$$

$$\sum_3 = 2 - 2 + 2 - 2 + 2 - 2 + \cdots$$

Let's consder the sum of the first n terms of the series. We will let the general temr of the series be denoted by t_i. This general term is the same as the general term for a geometric sequence. So, $t_i = ar^i$, $i = 0, 1, 2, ldots$. Now the sum of the first n terms is,

$$
\begin{aligned}
S_n &= t_0 + t_1 + \cdots + t_n \\
&= a + nar + ar^2 + \cdots + ar^n \\
&= a(1 + r + r^2 + \cdots + r^n) \\
&= a\left(\frac{1 - r^n}{1 - r}\right)
\end{aligned}
$$

Note: $1 + r + r^2 + \cdots + r^n = \frac{1 - r^n}{1 - r}$, $r \neq 1$. So,

$$S_n = a\left(\frac{1 - r^n}{1 - r}\right) \text{ or} \tag{1}$$

$$S_n = \frac{a - t_n}{1 - r} \tag{2}$$

Exercises

1. Determine which series are arithmetic.

   a) $1 + 2 + 3 + \cdots$

   b) $3 + 5 + 7 + 9 + 11 + \cdots$

   c) $1 + 2 + 4 + 8 + 16 + \cdots$

   d) $1 + 3/2 + 2 + 5/2 + 3 + 7/2 + \cdots$

   e) $-1 + 1 - 1 + 1 - 1 \cdots$

   f) $3 + 3/2 + 3/4 + 3/8 + 3/16 + \cdots$

   g) $4 + 0 - 4 - 8 - 12 - \cdots$

   h) $3 - 6 + 12 - 24 + 48 - 96 + \cdots$

   i) $1/3 + 1 + 3 + 9 + 27 + 81 + \cdots$

   j) $1 + 3 + 5 + 9 + 17 + 33 + \cdots$

   k) $6 + 11 + 16 + 21 + 26 + \cdots$

2. For the geometric series in # state the general term, e.g. $t_n = ar^n$.

3. For the geometric series in #1, find the sum S_n.

4. Find the following sum,

$$\sum_{n=1}^{7} 2^{n-1}$$

5.
$$\frac{1}{2} + \frac{7}{4} + \frac{49}{8} + \cdots + \frac{2401}{32} = ?$$

6. Find the sum of the first 8 terms of,

$$3, 6, 12, \ldots$$

7. Find the sume of,

$$S_1 = 1 - \frac{1}{\sqrt{2}} + \frac{1}{2} - \frac{1}{2\sqrt{2}} + \frac{1}{4} - \frac{1}{4\sqrt{2}} + \frac{1}{8}$$

8. Given the information below, find a_3, a_6 and a_9 for the geometric sequence,

a) $a_1 = 8, \ a_4 = 320$

d) $a_4 = \frac{1}{9}, \ r = -\frac{1}{3}$

b) $a_4 = -\frac{1}{9}, \ r = \frac{1}{3}$

e) $a_7 = 729, \ r = 3$

c) $a_5 = 48, \ a_6 = 96$

f) $a_3 = 18, \ a_5 = 162$

g) $a_7 = -2$, $r = -\frac{1}{3}$

i) $a_3 = -1$, $a_7 = \frac{1}{16}$

h) $a_4 - a_2 = 8$, $a_2 + a_3 = 4$

j) $a_5 = 4a_3$, $a_3 + a_0 = 63$

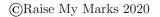9. Find the sume S_10 for the geometric series given the information below,

a) $S_n = 2^n$

e) $a_1 = 2$, $r = 3$

b) $a_4 = 4$, $a_8 = 16$

f) $r = 2$, $S_7 = 508$

c) $a_2 = 4$, $a_3 = -8$

g) $5a_2 - a_4 = 24$, $a_5 = 9_3$

d) $a_1 = -1$, $r = 0.3$

h) $a_2 = \frac{3}{2}$, $a_7 = -\frac{3}{64}$

10. The finite geometric sequence has 10 terms. The sum of all the terms with even index is 682 and the sum of the all terms with odd index is 1364. Determine the first term and quotient, r, of the sequence.

11. The sum of a_1 and a_3 is 15. The sum of the first 3 terms of the sequence is 21. Determine the first term and the quotient, r.