Compound Interest

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What is Compound Interest?

Simple interest is the interest that is calculated on a fixed amount at the end of a period of time. For example, if you have $P = \$1000$ invested in the bank for 3 years at an annual interest rate of 4% the interested earned at the end of the 3 years would be,

$$\begin{aligned} I &= Prt \\ &= (\$1000)(0.04)(3) \\ &= \$120 \end{aligned}$$

So, \$120 in interest was earned over the 3 year period. The total amount that you now have after that 3 year period is,

$$A = P + I = \$1000 + \$120 = \$1120$$

Compound interest is when that given annual interest rate r is calculuated for the year but at equal intervals within the year. So for example, if the annual interest rate is still 4% but now instead of calculating the interest at the end of each year, you calculate the interest earned each month. Let's take a closer look at this over a one year period.

Month	Amount	Interest earned
0	1000	$(1000)(0.04/12) = 3.33$
1	$1000 + 3.33 = 1003.33$	3.34
2	1006.67	3.36
3	1010.03	3.37
4	1013.40	3.38
5	1016.78	3.39
6	1020.17	3.40
7	1023.57	3.41
8	1026.98	3.42
9	1030.40	3.43
10	1033.83	3.45
11	1037.28	3.46
12	1040.74	

Let's take a look at this table. Starting with month 0 means this is the amount we are starting with so our principal. The interest earned in the first month is calculated with the following formula,

$$\begin{aligned} \text{Interested earned} &= P * \frac{r}{n} \\ I_1 &= 1000 * \frac{0.04}{12} \end{aligned}$$

The amount at the end of the first month is now given by

$$\begin{aligned} A_1 &= P + I_1 \\ &= 1000 + 1000 * \frac{0.4}{12} \\ &= 1003.33 \end{aligned}$$

This is the amount we calculate the second month's interest on. The interest rate for the second month is the same as that for the first month which was given by,

$$\begin{aligned} i &= \frac{r}{n} && (1) \\ &= \frac{0.04}{12} = 0.0033 && (2) \end{aligned}$$

Here r is the interest rate given for the year; n is the number of period per year that that interest is calculated. In this case since the interest is calculated monthly, $n = 12$ since there are 12 months in a year. Continuing in this way we see that after 1 year total amount A that was have is \$1040.74 and the interest earned over this year is,

$$\begin{aligned} I &= A - P && (3) \\ &= 1040.74 - 1000 && (4) \\ &= \$40.74 && (5) \end{aligned}$$

If we compare this amount to the amount of interest that would have earned if we used simple interest, which would be $\$1000*0.04 = \$40$, we gained an extra \$0.74 over that year using compound interest rather than simple interest. Doesn't seem llike much. But for investments over a longer period of time t, the gain from compound interest could be very significant. Let's define some variables associated with some of the values that we used in the above calculations for compound interest.

Variable	Name	Description
P	Principal	Initial amount invested
A	Total earned	The total amount earned at the end of the term. $A = P(1+i)^{nt}$
t	investment period	time period over which principal is invested
r	annual interest rate	interest rate for the year
n	compounding period	number of times the interest is calculated per year
i	interest calculated per compounding period	$i = \frac{r}{n}$

In our example above the frequency or compounding period over which the interest is calculated was every month. However, it is not always the case that this frequency is monthly. It could be every day, twice a year, every othr week, anything. Below is a table of the more common period over which interest is calculated tand the corresponding n value or number of compounding periods per year.

Period	n
annual	1
semi-annual	2
quarterly	4
monthly	12
bi-weekly	26
weekly	52
daily	365

Sometimes we want a specific final amount A after a specified investement period t. We are given an interest rate r and the compounding period n. What we don't know is how much we have invcest today or in the *present value* of this final investment. The present value is given by the following formula,

$$PV = \frac{A}{(1+i)^{nt}} \tag{6}$$

Exercises

1. Find the following amounts,
   a) \$2000 invested at 5%/ year compounded quarterly for 6 years.
   b) \$2250 invested at 4.25%/ year compounded daily for 9 years.
   c) \$3500 invested at 6%/ year compounded quarterly for 12 years.
   d) \$4000 invested at 7%/ year compounded semi-annually for 4 years.
2. For each of the final amounts in # 1, what is the interest earned?
3. Suppose you are saving up for a car and need at least \$10000 to buy a car. You want to buy the ar at the nd of high school so you'll need the money in 4 years. You decide to invest money in an account with an interest rate of 5%/year compounded bi-weekly. How much do you need to put into the account now to be able to buy a car in 4 years?
4. A \$500 savings bond pays 6.5% compounded semi-annually and matures in 7.5 years.
   a) What is the purchase price?
   b) How much will be earned?
   c) What is the final amount?
5. A \$10000 Treasure Bill earned 16% compounded monthly. If the bill matures in 2 years, what is it worth at maturity?
6. What principal will amount to \$2500 if invested at 5%4 interest compouneded quarterly for 8 years?