Compound Interest

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What is Compound Interest?

Simple interest is the interest that is calculated on a fixed amount at the end of a period of time. For example, if you have $P = \$1000$ invested in the bank for 3 years at an annual interest rate of 4% the interested earned at the end of the 3 years would be,

$$\begin{aligned} I &= Prt \\ &= (\$1000)(0.04)(3) \\ &= \$120 \end{aligned}$$

So, \$120 in interest was earned over the 3 year period. The total amount that you now have after that 3 year period is,

$$A = P + I = \$1000 + \$120 = \$1120$$

Compound interest is when that given annual interest rate r is calculuated for the year but at equal intervals within the year. So for example, if the annual interest rate is still 4% but now instead of calculating the interest at the end of each year, you calculate the interest earned each month. Let's take a closer look at this over a one year period.

Month	Amount	Interest earned
0	1000	$(1000)(0.04/12) = 3.33$
1	$1000 + 3.33 = 1003.33$	3.34
2	1006.67	3.36
3	1010.03	3.37
4	1013.40	3.38
5	1016.78	3.39
6	1020.17	3.40
7	1023.57	3.41
8	1026.98	3.42
9	1030.40	3.43
10	1033.83	3.45
11	1037.28	3.46
12	1040.74	

Let's take a look at this table. Starting with month 0 means this is the amount we are starting with so our principal. The interest earned in the first month is calculated

with the following formula,

$$\begin{aligned} \text{Interested earned} &= P * \frac{r}{n} \\ I_1 &= 1000 * \frac{0.04}{12} \end{aligned}$$

The amount at the end of the first month is now given by

$$\begin{aligned} A_1 &= P + I_1 \\ &= 1000 + 1000 * \frac{0.4}{12} \\ &= 1003.33 \end{aligned}$$

This is the amount we calculate the second month's interest on. The interest rate for the second month is the same as that for the first month which was given by,

$$\begin{aligned} i &= \frac{r}{n} && (1) \\ &= \frac{0.04}{12} = 0.0033 && (2) \end{aligned}$$

Here r is the interest rate given for the year; n is the number of period per year that that interest is calculated. In this case since the interest is calculated monthly, $n = 12$ since there are 12 months in a year. Continuing in this way we see that after 1 year total amount A that was have is \$1040.74 and the interest earned over this year is,

$$\begin{aligned} I &= A - P && (3) \\ &= 1040.74 - 1000 && (4) \\ &= \$40.74 && (5) \end{aligned}$$

If we compare this amount to the amount of interest that would have earned if we used simple interest, which would be $\$1000 * 0.04 = \$40$, we gained an extra \$0.74 over that year using compound interest rather than simple interest. Doesn't seem llike much. But for investments over a longer period of time t, the gain from compound interest could be very significant. Let's define some variables associated with some of the values that we used in the above calculations for compound interest.

Variable	Name	Description
P	Principal	Initial amount invested
A	Total earned	The total amount earned at the end of the term. $A = P(1+i)^{nt}$
t	investment period	time period over which principal is invested
r	annual interest rate	interest rate for the year
n	compounding period	number of times the interest is calculated per year
i	interest calculated per compounding period	$i = \frac{r}{n}$

In our example above the frequency or compounding period over which the interest is calculated was every month. However, it is not always the case that this frequency is monthly. It could be every day, twice a year, every othr week, anything. Below is a table of the more common period over which interest is calculated tand the corresponding n value or number of compounding periods per year.

Period	n
annual	1
semi-annual	2
quarterly	4
monthly	12
bi-weekly	26
weekly	52
daily	365

Sometimes we want a specific final amount A after a specified investement period t. We are given an interest rate r and the compounding period n. What we don't know is how much we have invcest today or in the *present value* of this final investment. The present value is given by the following formula,

$$PV = \frac{A}{(1+i)^{nt}} \tag{6}$$

Exercises

1. Find the following amounts,
   a) $1500 invested at 4%/ year compounded monthly for 7 years.
   b) $900 invested at 6%/ year compounded semi-annually for 5 years.
   c) $950 invested at 3.75%/ year compounded bi-weekly for 12 years.
2. For each of the final amounts in # 1, what is the interest earned?
3. What principal do you have to deposit in a 4.5% savings account compounded monthly in order to have a total of $10000 after 8 years?
4. $1000 is deposited into each of two bank accounts A and B. Account A has a 3%/year interest rate, simple interest. Account B also has a 3%/year interest rate but compounded annually. After 3 years how much money is in each account?
5. Say you invest your birthday money of $100 in a savings acocunt compounded montly at a rate of 4.25% / year. After 4 years, how uch money do you have?