Composition of Functions 4

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What is the composition of functions?

The composition of functions means, one function is inserted into the another funcionn where a variable would normally go. If we consider functions $f(x)$ and $g(x)$, the composition of two functions f and g means, the function g is inserted into the function f as the value of x, or $x = g(t)$ for $f(x)$. Let's a look at an example to get a better idea of what this means and looks like.

Example

Let's look at the compostion of two plynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \text{ and } g(t) = t + 1.$$

What is the composition of f and g or in notation, what is $f \circ g$?

Solution The compoistion of f and g looks like,

$$f \circ g(t) \text{ or } f(g(t))$$

and is given by,

$$\begin{aligned} f(g(t)) &= f(t+1), \text{ where } x = g(t) = t + 1 \\ &= (t+1)^2 + 3(t+1) - 1 \\ &= (t^2 + 2t + 1) + (3t + 3) - 1 \\ &= t^2 + 2t + 1 + 3t + 3 - 1 \\ \therefore f(g(t)) &= t^2 + 5t + 3 \end{aligned}$$

is the resulting polynomial.
Let's try another example.

Example

Given $f(x) = 2 - x$ and $g(x) = \frac{2}{5-x}$ determine $f \circ g(x)$ and $g \circ f(x)$.

Solution: Let's start with $f \circ g(x)$.

$$\begin{aligned} f(g(x)) &= f\left(\frac{2}{5-x}\right) = 2 - \frac{2}{5-x} = \frac{2(5-x)-2}{5-x} \\ &= \frac{10-2x-2}{5-x} = \frac{8-2x}{5-x} \end{aligned}$$

Therefore, $f(g(x)) = \frac{8-2x}{5-x}$.

Now $g \circ f(x)$.

$$g \circ f(x) \quad = \quad g(2-x) = \frac{2}{5-(2-x)} = \frac{2}{5-2+x} = \frac{2}{3+x}$$

Therefore, $g(f(x)) = \frac{2}{3+x}$.

Exercises

1. Given $f(x) = 2 - x^2$ and $g(x) = -3x$, determine the following values,

   a) $f(-1)$

   b) $g \circ f(-1)$

   c) $g(0)$

   d) $f(g(0))$

   e) $f(2)$

   f) $g(f(2))$

   g) $f \circ g(-2)$

2. For the following pairs of functions determine $f \circ g(x)$ and $g(f(x))$.

   a) $f(x) = 2 - x^2,\ g(x) = 4x + 3$

   b) $f(x) - x^2 + 1,\ g(x)3 - x$

3. For each pair of funcations in #4 determine,

   a) the domain of $f \circ g$

   b) the range of $f \circ g$.