Composition of Functions 3

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2020

What is the composition of functions?

The composition of functions means, one function is inserted into the another funcionn where a variable would normally go. If we consider functions $f(x)$ and $g(x)$, the composition of two functions f and g means, the function g is inserted into the function f as the value of x, or $x = g(t)$ for $f(x)$. Let's a look at an example to get a better idea of what this means and looks like.

Example

Let's look at the compostion of two plynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \ \text{and} \ g(t) = t + 1.$$

What is the composition of f and g or in notation, what is $f \circ g$?

Solution   The compoistion of f and g looks like,

$$f \circ g(t) \ \text{or} \ f(g(t))$$

and is given by,

$$
\begin{aligned}
f(g(t)) &= f(t+1), \ \text{where } x = g(t) = t + 1 \\
&= (t+1)^2 + 3(t+1) - 1 \\
&= (t^2 + 2t + 1) + (3t + 3) - 1 \\
&= t^2 + 2t + 1 + 3t + 3 - 1 \\
\therefore f(g(t)) &= t^2 + 5t + 3
\end{aligned}
$$

is the resulting polynomial.
Let's try another example.

Example

Given $f(x) = 2 - x$ and $g(x) = \frac{2}{5-x}$ determine $f \circ g(x)$ and $g \circ f(x)$.

Solution: Let's start with $f \circ g(x)$.

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{2}{5-x}\right) = 2 - \frac{2}{5-x} = \frac{2(5-x) - 2}{5-x} \\
&= \frac{10 - 2x - 2}{5-x} = \frac{8 - 2x}{5-x}
\end{aligned}
$$

Therefore, $f(g(x)) = \frac{8-2x}{5-x}$.

Now $g \circ f(x)$.

$$g \circ f(x) \quad = \quad g(2-x) = \frac{2}{5-(2-x)} = \frac{2}{5-2+x} = \frac{2}{3+x}$$

Therefore, $g(f(x)) = \frac{2}{3+x}$.

Exercises

1. Given $f(x) = x^2 - x$ and $g(x) = 4x + 1$,

   a) find $f(g(x))$ and

   b) find $g(f(x))$.

2. For each function determine f and g so that $y = f(g(x))$.

   a) $y = (x+1)^2 - 2x(x+1) + 3$

   b) $y = \frac{1}{2x+3}$

   c) $y = -2\sqrt{x-1}$

   d) $y = (x^2 + 3)^4$

   e) $y = -(4x - 5)^3$

   f) $y = \sqrt{x+2}$

   g) $y = -\frac{1}{x+1}$

   h) $y = (x+1)^2$

   i) $y = \sqrt{x^2 + 1}$