

Composition of Functions

**Raise My**  
**MArks**

RaiseMyMarks.com

2020

## What is the composition of functions?

The composition of functions means, one function is inserted into the another function where a variable would normally go. If we consider functions  $f(x)$  and  $g(x)$ , the composition of two functions  $f$  and  $g$  means, the function  $g$  is inserted into the function  $f$  as the value of  $x$ , or  $x = g(t)$  for  $f(x)$ . Let's look at an example to get a better idea of what this means and looks like.

### Example

Let's look at the composition of two polynomials. For example, let's consider

$$f(x) = x^2 + 3x - 1 \text{ and } g(t) = t + 1.$$

What is the composition of  $f$  and  $g$  or in notation, what is  $f \circ g$ ?

**Solution** The composition of  $f$  and  $g$  looks like,

$$f \circ g(t) \text{ or } f(g(t))$$

and is given by,

$$\begin{aligned} f(g(t)) &= f(t + 1), \text{ where } x = g(t) = t + 1 \\ &= (t + 1)^2 + 3(t + 1) - 1 \\ &= (t^2 + 2t + 1) + (3t + 3) - 1 \\ &= t^2 + 2t + 1 + 3t + 3 - 1 \\ \therefore f(g(t)) &= t^2 + 5t + 3 \end{aligned}$$

is the resulting polynomial.  
Let's try another example.

### Example

Consider,

$$f(x) = 3x^3 + 2x \text{ and } g(x) = -2x^2 + 3$$

What is  $g \circ f(x)$ ?

**Solution:** Then the composition of  $g \circ f(x)$  or  $g(f(x))$  is given by,

$$\begin{aligned}g(f(x)) &= g(3x^3 + 2) \\&= -2(3x^3 + 2)^2 + 3 \\&= -2((3x^3)^2 + 2(2)(3x^3) + 4) + 3 \\&= -2(9x^6 + 12x^3 + 4) + 3 \\&= -18x^6 - 24x^3 - 8 + 3 \\ \therefore g(f(x)) &= -18x^6 - 24x^3 - 5\end{aligned}$$

## Exercises

Evaluate  $f \circ g(x)$  where  $f$  and  $g$  are given below.

a)  $f(x) = 3x^2 + 2$  and  $g(x) = 4x + 3$

b)  $f(x) = x^2 + 3x + 2$  and  $g(x) = 2x^2 + 1$

c)  $f(x) = x^3 + 2x - 4$  and  $g(x) = -3x + 1$

d)  $f(x) = x^6 + x^4 + 1$  and  $g(x) = x^2 - 2$

e)  $f(x) = x^5 + 3x^2 + x - 1$  and  $g(x) = x - 1$