Chain Rule 2

RaiseMyMarks.com

2020

Chain Rule - Differentiation

Given functions $f(x)$ and $g(x)$ the deritivate of the function,

$$F(x) = f(g(x)) = f \circ g(x)$$

is given by,

$$F'(x) = f'(g(x))g'(x) \tag{1}$$

Equation (1) is called the *chain rule* of differentiation.

Example

Find the derivative of the function,

$$y = (x^3 + 1)^2 \tag{2}$$

Solution: We first need to determine what the two function f and g are in order to apply the chain rule (1). Taking a look at equation (2) if we take $f(x) = x^2$ and $g(x) = x^3 + 1$, the composition $f \circ g(x)$ gives us the function $f \circ g(x) = (x^3 + 1)^2$. So, now we can apply the chain rule and we know what functions f and g we are considering in equation (1).

$$\begin{aligned} y' &= 2(x^3 + 1)^1(3x^2) \\ &= 2(3x^2)(x^3 + 1) \\ &= 6x^2(x^3 + 1) \end{aligned}$$

Therefore, $y' = 6x^2(x^3 + 1)$.

Exercises

Use the chain rule to differentiate the following functions.

a) $y = \sqrt{2 - x + x^2}$

b) $y = -\sqrt{3 - 2x^2}$

c) $y = \sqrt{2x + x^4}$

d) $y = \left(\frac{x^2+1}{x+1}\right)$

e) $y = \left(\frac{1+x+x^2}{1-x-x^2}\right)$

f) $y = \left(\frac{2-x^2}{2+x^3}\right)^3$

g) $y = (2x^2 - 3x)^5$

h) $y = \frac{1}{(x^2-3)^4}$